1. Introduction

Photonic Crystal structures attract much attention as new Optical materials whose permittivity periodically changes in one, two, or three dimensions with the spatial scale comparable with the light wavelength [1,2]. These crystals were proposed for the first time in 1987 [3] and the first photonic crystal was realized in 1991 by Yablonovitch [4]. The importance of these structures is the presence of photonic band gaps and allowed bands that permits a strip, which corresponds to a mode that propagates in the photonic crystal. A band gap corresponding to an energy range or the propagation of light is prohibited in certain directions of the photonic crystal. These crystals may have applications in the telecommunication optical fibre [5], sensors [6], the infrared filters [7], absorbers graphenes [8], other photonic dispositive [9], optical filters and photonic-based devices [10], optical isolators and circulators and modulators [11], guides and optical cavities [12], they can also be used to make integrated circuits [13].

The creation of the defects in photonic crystals allows the creation of status, which are permitted in the gap. These status are called also the modes of defects where the light can spread through the gap, these modes appear like the maximum of the spectrum of transmission and they depend on the nature of layers defects, thickness and the positions of defects, these crystals which contain the defects have attracted considerable attention. In reality, photonic structures, generally, have artificial or natural defects representing periodicity of disturbances within these structures. The creation of defects in the photonic crystals is done either by the local modification on the refraction index, hence called material defects, or by changing the size of a material then called geometrical defects. For photonic crystals that are composed of dielectric materials, the refractive index of a material can be changed; thereby creating a defect in the structure is called default materials.

Several teams studied the defects materials [10,14]. B.Xu showed that the variation of the thickness and the index refraction of layer defects might change the position and the width of the gap [10]. X. Xiao studied the optical properties of one-dimensional photonic crystal filters with one Al₂O₃ or TiO₂ defect layer both theoretically and experimentally. They showed that the lower refractive index defect layer Al₂O₃ leads to a stronger transmission resonance. And they found that, as the number of period’s increases; the filter resonance becomes weaker and eventually vanishes [14]. Several teams have also studied the geometrical defects [7-9,15,16]. Experimentally, Bipin and al showed the existence of two defect modes inside the photonic band gap for the quarter-wave layers stacking photonic crystal structures with a defect layer of graded index material and the intensity, frequency and number of defect modes have been changed with the thickness and gradation parameter of graded index defect.
layer[9]. V. A. Guenyakov could tune gradually the transmittance spectrum of photonic crystal by varying the applied voltage on a multilayer photonic crystal with a nematic liquid crystal layer. In addition, the spectrum strongly depends on the light polarization direction above threshold voltage. The application of an alternating electric field on this crystal is used to change the transmittance spectrum [15]. Y.J. Liu et al showed that for Crystal Photonics containing two defects dielectric supraconductors, peaks of absorptions increase with the decrease of the thickness of layers defects and the variation of the incidence angle adjusts the peak absorptions [8]. C.C. Liu studied the infrared defect mode in defective photonic crystal made of SiO₂ and InP and using of ITO as a defect. They found that the defect mode as sensitive to the ITO thickness and the defect frequency is shown to be blue-shifted as the thickness of ITO decreases. The shift in the defect frequency enables us to employ ITO as a tunable agent in order to design a tunable photonic crystal filter in the infrared region [7]. N.Ouchani and D. Bria showed that defect modes of anisotropic photonic crystal appear in the transmission spectrum. The number, the intensity and the frequency are extremely sensitive to the incidence angle, the position of layers defects and the orientation of the principal axes layers Crystal. It may be a good candidate for the realization of electromagnetic selective filter [16]. A. Bousfia showed that creating a geometric defect of a photonic crystal can lead to a total reflection of waves at all angles and polarizations incidents in a given frequency range "Omnigap", and allows to find selective wave in this interval which may be transmitted through the structure [17]. A. Mir showed that for a one-dimensional periodic structure containing a defect could obtain localized modes within these band gaps[18]. A. Aghajamali showed that for a photonic crystal containing geometrical defects, application of electric and magnetic fields on these crystals can change the transmission curves [19]. The theory of Green function [20], was recently presented to some composite materials in discrete areas where there are continuous spaces [21].

The method of Green's function calculation allows us to deduce the coefficients of reflection and transmission and relationship dispersions. This theory has been the subject of many applications in the fields of electronics [21], dissipative quantum systems [22] magneto-electroelastic [23] and magnetism [24], and considerable physical information [25]. Knowledge of the constituents of each response function of a composite material allows using the interface response theory [26-27] to deduce the dispersion relations of the eigen modes, the expressions of the corresponding eigenvectors, the densities of local, total states, the coefficients of transmission and reflection of any composite type [28].

The present paper presents theoretical and numerical studies of the coefficient of transmission and reflection of double-defective photonic crystal (PC) constituted of two alternating layers of respective refractive index n₁ and n₂ and respective thicknesses d₁ and d₂, containing double defect layers. The theoretical calculations based on the formalism of Green's function for the electromagnetic waves propagating in periodic layered media.

2. Model and formalism

The geometrical structure of double-defective photonic crystal (PC) considered is schematically shown in figure1. The defect layers labelled “01” and “02”, of thicknesses d₀₁ and d₀₂, and refraction index n₀₁ and n₀₂, are inserted in the perfect PC structure at the positions j and j’, respectively. The unit cell of the structure is composed of two isotropic materials of refractive index nᵢ (material i=1) and of refractive index nᵢ (material i=2). The materials constituting the layers (i=01,02,1,2) of the structure are assumed homogeneous, nonmagnetic and characterized by their thicknesses d₀₁,d₀₂,d₁ and d₂, respectively. The dielectric constants of material i are εᵢ. All the interfaces are taken to be parallel to the (x₁,x₃) plane of a Cartesian coordinate system and the x₃ axis is along the normal to the interfaces. The number of the period of the perfect PC is N and its period is D=d₁+d₂.

Figure1: Geometrical structure of double-defective photonic crystal (PC). j and j’ are respectively the position of the defect layer 01 and 02 of thicknesses d₀₁ and d₀₂ and refraction index n₀₁ and n₀₂, respectively.
In this paper, we developed the efficient model called the interface response theory to study the optical waves in lamellar periodic structures. The object of this theory is to calculate the Green’s Function of a composite system containing a large number of interfaces that separate different homogenous media. The knowledge of this Green’s function enables us to obtain different physical properties of the system such as the reflection and transmission coefficients of the waves. In this theory the Green’s function of a composite system can be written as [29]:

\[ g(DD) = G(DD) + G(DM) \left\{ G(MM) \right\}^{-1} g(MM) \left\{ G(MM) \right\}^{-1} \left\{ G(MM) \right\}^{-1} G(MD) \] (1)

Where D and M are respectively the whole space and the space of the interfaces in the lamellar system. G is a block-diagonal matrix which each sub-block \( G_i \) correspond to the bulk Green’s function of the subsystem i. One can notice that all the matrix elements \( g(DD) \) of the composite material can be obtained from the knowledge of the matrix elements \( g(MM) \) of the g in the interface space M. The latter are obtained by writing the matrix \( g^{-1}(MM) \) as a sum of the submatrices \( g_i^{-1}(MM) \) in each sub-block \( i \) considered separately. All details about the analytical calculation of the Green’s function for finite layer as well as infinite superlattice can be found in Ref[30]. Within this theory the reflected and transmitted waves \( u(D) \) resulting when a uniform plane wave incident \( U(D) \) upon a plane boundary between two different media are given by [29]:

\[ u(D) = U(D) + G(DM) \left\{ G(MM) \right\}^{-1} g(MM) \left\{ G(MM) \right\}^{-1} \left\{ G(MM) \right\}^{-1} U(M) \] (2)

3. Results and discussions

3.1. Geometric defects

We consider the substrate structure Air / crystal photonic (\( SiO_2/GaAs \) ) / Air With two layers of defects inside the structure. Our crystal contains nine layers of \( GaAs \) (material \( i=2 \)) and ten layers of \( SiO_2 \) (material \( i=1 \)). The thicknesses of the layers are respectively \( d_01 = 0.4Dd_2 = 0.6D \). The refractive index of the two media is respectively \( n_1 = 1.47 \) and \( n_2 = 1.87 \). The two defects have the same refractive index \( n_{01} = n_{02} = 1.50 \), whose thicknesses are noted \( d_{01} \) and \( d_{02} \). We investigate and discuss The structure of the band gap with the various parameters of the thickness \( d_{01} \) and \( d_{02} \), the positions \( j_{def1} \) and \( j_{def2} \) the two defects layers.

3.1.1. Photonic band gap structure as a function of the parameters \( d_{01} \) and \( d_{02} \)

In the following, we investigate the possible behaviours of TE and TM modes transmission curves by choosing different values of the parameters \( d_{01} \) and \( d_{02} \). The first defect is in cell 4 and the second one is considered in cell 6. An electromagnetic mode that is introduced by the geometrical defects is called geometric defect modes. These modes whose frequencies can be in the band gap and be located around the defect site. Figure 2 shows the evolution of the transmission as a function of reduced frequency for different values of the thicknesses of the defects \( d_{01} \) and \( d_{02} \). We observe that when \( d_{01} = d_{02} = 0 \) (case a), there are two peaks in the band gap with a transmission value \( t = 0.8 \). On the contrary, in the case b, when \( d_{01} = d_{02} = d_1/2 \), there is a single peak in the band gap with a transmission value \( t = 0.8 \). Again, in the case c, when \( d_{01} = d_{02} = d_1 \), there are two peaks in the band gap with the same transmission value \( t = 0.9 \). When the value of the thickness of the defects increases, there are always two peaks in the band gap with one gap with transmission value \( t = 0.9 \) and the other with transmission value \( t = 0.8 \) when \( d_{01} = d_{02} = 2d_1 \) (case d) and with the same transmission value \( t = 0.9 \) when \( d_{01} = d_{02} = 3d_1 \) (case e). These two peaks in the band gap move away from each other with an increasing of the thickness of the defects \( d_{01} \) and \( d_{02} \).

3.1.2. Photonic band gap structure as a function of the parameters \( j_{def1} \) and \( j_{def2} \)

Now we are interested in the possible behaviours of TM and TE modes transmission curves by choosing different values of the parameters \( j_{def1} \) and \( j_{def2} \). The other parameters are taken to be \( d_{01} = d_{02} = 2d_1 \). Figure 3 shows the evolution of the transmission as a function of reduced frequency for different positions of two defects \( d_{01} = d_{02} = 2d_1 \). For different positions of defects, we observe that when the two defects are very away from each other (case e), there is a single peak in the band gap (overlap) with a transmission value \( t = 0.8 \). On the other hand, when approaching these two defects (case d), we notice that there’s a superposition of two peaks in the gap with significant transmission value \( t = 0.8 \) and low frequency equal to 0.28. For the (case c), we notice that the superposition of the two peaks begins to separate and they are obtained in the band gap with a transmission value \( t = 0.8 \). These two peaks continue to move away with a transmission value \( t = 1 \) when both defects continue to approach (case b). For the case (a), and in the same way, we note that the two separate peaks are located in the band gap but they depart together with a transmission value \( t = 0.8 \). From comparison to
various cases of figure 3, we conclude that the merger of the two defects is proportional to the distances of the peaks.

**Figure 2:** Variation of transmission as a function of the reduced frequency for different values of $d_{01}$ and $d_{02}$.

**Figure 3:** Variation of transmission as a function of the reduced frequency for different positions of two defects for $d_{01} = d_{02} = 2d_1$. 
3.1.3. Photonic band gap structure as a function of the parameters $d_{01}$ and $j_{def1}$

In this section, we examine the effect of thickness and the position of the two defects on the frequency of the modes located in the band gap. We are talking about the influence of the thickness and the defect position on the gap modes. For the first part, we are interested in the possible behaviours of TM and TE modes dispersion curves by choosing different values of the parameter $d_{01}, j_{def1}$ and $j_{def2}$. The other parameter is taken to be $d_{02} = 3d_2$, and in the second part, we are interested in the possible behaviours of TM and TE modes dispersion curves by choosing different values of the parameter $d_{01}, j_{def1}$ and $j_{def2}$. The other parameter is taken to be $d_{02} = 3d_2$.

Now, we study the effect of the thickness and position of the two defects on the reduced frequency of modes in the band gap. Figure 4 and 5 give the dispersions curves of the defects modes in the band gap. Figure 4 shows the variation of the maximum reduced frequency, the function of the thickness of the first defect $d_0$ with $d_{02} = 3d_2$.

**Figure 4:** Variation of the maximum reduced frequency as a function of the thickness of the first defect $d_0$ with $d_{02} = 3d_2$
Figure 5: variation of the maximum reduced frequency as a function of the position of the first defect \(d_{01}\) for different thickness of \(d_{01}\) and \(d_{02}\). We notice that when \(d_{01} = d_{02} = d_1/2\), the absence of the peaks in the band gap (case a). Against by, for \(d_{01} = d_{02} = d_3\) (case b) we notice the appearance of two peaks in band gap. The distance between the two peaks varies in proportion to the increase of the position of the defect. Similarly for \(d_{01} = d_{02} = 3d_2\) (case c), with the existence of the two peaks in the middle of the gap. We conclude that the best thickness of defect is \(d_{01} = d_{02} = 3d_2\).

Conclusions
Numerically, we have studied a photonic structure composed by two respective index materials \(n_1=1.49\) and \(n_2=1.87\) and the respective thicknesses \(d_1=0.4\) and \(d_2=0.6\). This structure is of period \(D=d_1+d_2\) and contains two similar defects layers. The refractive index of this two cavities are \(n_{01} = n_{02} = 1.5\) and their thicknesses are \(d_{01} = d_{02} = 1.5\). We conclude that after the transmission of the curves, the number of modes of defects, according to the number of modes of defects depends on the defects thicknesses. This study shows that when two defects are far
from each other there is a single mode in the gap. And when they are approached together they continue to approach again. We notice that the appearance of the two modes in the gap is receding when the two defects continue to approach together. We notice that when the two defects are far away from each other, a superposition of two branches appears. And when they get closer, they show two branches which are well localized in the gap.

References