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# Model of Mating Of Parts with Nominal Coaxial Surfaces of Rotation

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## Abstract
Proposed model allows determining the relative position of the mating parts on nominally coaxial surfaces of rotation. The model is based on the algorithm of the nearest points (ICP), which simulates rigid contact of mating surfaces. To determine the possible positions of the mating surfaces in the model assembly conditions are simulate, they are determined by the vector assembly force. Uncertainties of mating pair of conical surfaces with the form deviation in the cross section are theoretically researched. Conclusions about the significant influence of the form deviation of the mating surfaces on the accuracy of the relative position of parts are obtained.

*Keywords:* uncertainty estimate, manufacturing, assembly, tolerancing.

## 1. Introduction
The quality of products and their performance parameters defined in the design are in large part determinedby a complex of measures on preparation for production. The measures include the design of manufacturing procedure, the development of those resources, engineering techniques, testing parameters and modes of processing, etc. As a rule, products consist of a plurality of individual parts, which are characterized by an ensemble of dimensions. As a result of the assembly of parts, the dimensional communication is formed determining the accuracy of the assembly parameters. Production errors occur in steps of individual components manufacturing and during the formation of the dimensional relations of parts in the assembly process. Precision of assembly parameters is one of the most important parameters that determine achieved quality of products.
In complex mechanical systems the precision of assembly parameters is the pacing factor of their performance. Fuel efficiency, propulsion, gas dynamic stability of aircraft engine, which is a complex mechanical system, largely depend on the size and non-uniformity of a radial clearance between blades tips and a stator. The radial clearance is formed by the size of such components as: an engine case, a disc, a blade, an operating ring.
Many sources [1,2] are concerned with manufacturing technology and assembly of aircraft engine with the use of different methods [3]. An integral element of manufacturing production is industrial gagging. Measurement by optical methods are widely used at present, it is characterized by high productivity [4]. Currently, the main method of process is machining process [5-7]. Techniques based on laser processing are widely used for modifying of materials and for repair [8-10].
Models of assembly taking into account the manufacturing errors are progressive area of research. Virtual assembly of complex systems taking into account the real dimensions of parts is considered in order to evaluate obtainable accuracy and optimize the dimension-accuracy parameters. The paper [11] shows the assembly of parts with deviation of surfaces position. It considered the assembly of parts mating flat to cylindrical surface and use linear programming to calculate the assembly parameters. To simulate mating pair of surfaces in [12] so-called Modal Coefficients are used for calculating the parameters of mating (points of contact and surface position). It is exemplified by mating two flat surfaces with the deviation relative to the nominal geometry.

A necessary condition of quality maintenance of industrial products is the evaluation and proved specification of tolerances. In the paper the model allowing estimating the uncertainty of three-dimensional mating of high-precision parts is shown.

## 2. Evaluation Method Of Uncertainty Of Three-Dimensional Mating Of Parts Surfaces

The mating of parts surfaces of complex systems is characterized by uncertainty, which is caused by the difference between the positions of parts surfaces. Uncertainty of parts surfaces mating may be caused by the form deviation of the mating surfaces relative to the nominal geometry due to influence of manufacturing errors as well as to variation of assembly conditions. The assembling conditions are determined by setting a fixed position of the first part and imposing vector of the assembly force applied to the second part. In the following sections we give a consequent description of the developed model.

### 2.1 Model of mating surface

The mating surfaces have a complex shape due to the presence of deviations. Curves and surfaces of complex shape are described by spline equations in CAD-systems and metrological supervision of measuring equipment. The spline is a piecewise polynomial to the power of K with a continuous derivative to the power of K-1 at the points of segments connection called a given point. For the mathematical representation of a complex surface we use a normalized cubic spline to 3 degrees, it is a Hermite curve described in [13].

To describe surfaces of parts with geometric deviation of form we use surface formed by bicubic patches (Coons patches). Described surface represents a segment corresponding to the parameters values $0 \le u \le 1$, $0 \le v \le 1$. The Coons patch is formed by the conjunction of boundary spline curves and it is defined by:

$$P(u,v) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} u^i v^j \ , \tag{1}$$

where $a_{ij}$ is an algebraic vector coefficients with constituent $x$, $y$ and $z$.

Combining of Koons patches makes possible to determine the surface of any shape and size. The spline surface is defined in parameter space $u$ and $v$.

### 2.2 The costate algorithm

The model of interfaces of curves or surfaces is based on the method of best fit with set of constraints on the intersection of surfaces. The interface of two surfaces with form deviations can be characterized by the size of the clearance G between the surfaces. In the case of the intersection of the surfaces the negative clearance is formed that is interference. Fig. 1 shows an example of a flat section of the spatial function describing the clearance between the two mating surfaces.

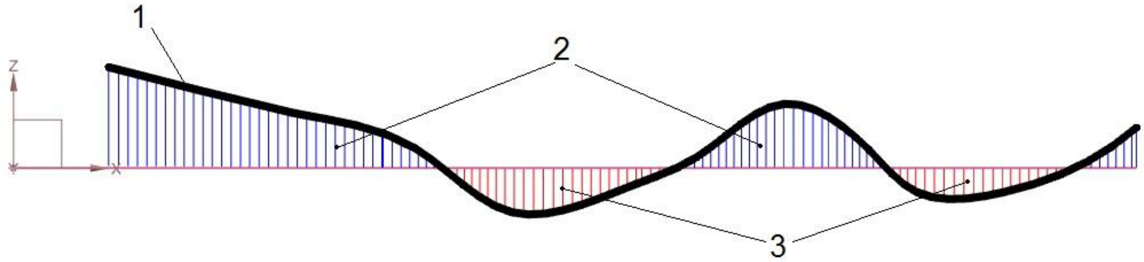


**Fig. (1).**An example of function describing the clearance between the two mating surfaces in a space

Each point of the spatial function is determined by the difference of coordinates of the mating surfaces points. If the clearance function (pos. 1 in figure 2) is located above the plane XOY (pos. 2 in Figure 2), there is a positive clearance between the surfaces. If the function is located below the plane XOY there is a negative clearance (item 3 in Figure 2.).

Depending on the $\overline{V}$ vector of relative surfaces position G function is used as an objective function characterizing the achievement of all the conditions of the mating surfaces of parts and it is defined as the sum of the distances between points, which produce negative clearance:

$$G(\overline{V}) = \sum d_{gap}; d_{gap} < 0 \, , \tag{2}$$

where $d_{gap}$ is the clearance.

In general the relative position vector includes three linear and three angular parameters for the respective axes. We developed an iterative algorithm to calculate the conjugation of parts with defined precision. The algorithm implements iterative movement of one surface with respect to the other along the coordinate axes. An iterative search is used for the displacement of one surface relative to the other until fulfillment of the condition:

$$\xi_1 \le -G(z) \le \xi_2, \tag{3}$$

where $\xi_1$ and $\xi_2$ are invariables specifying error of the costate.

Consider the implementation of the algorithm for finding the costate in the case of movement of one surface with respect to the other along the z axis wherein the application vector of assembly force equals $\overline{D_1} = (0,0,1)$.

Selection of z is carried out by the uniform search method with optimum localization. To calculate the function G (z) the best fit of surfaces in the plane XOY is performed at each stage. A common algorithm for solving the problem of the best fit is an iterative algorithm of the nearest points (ICP) presented in [14]. According to this algorithm at each iteration of the search methods of nonlinear optimization rotation angles and displacements along the coordinate axes are calculated. When a detail displaces in the plane XOY the objective function of algorithm can be defined as follows:

$$f(R,t) = \frac{1}{n_p}\sum_{ip=1}^{N}\left\|\overline{p}_{пов2\_ip} + \overline{T} - \overline{p}_{пов1\_ip}\right\|^2 \to 0, \tag{4}$$

where $n_p$ is the number of matching points;

$\overline{p}_{пов1\_ip}$ is the vector of the first surface coordinates;

$\overline{T}$ is the displacement vector;

$\overline{p}_{пов2\_ip}$ is the vector of the second surface coordinates corresponding to $\overline{p}_{пов1\_ip}$.

To search for function parameters (4) we apply the method of sequential quadratic programming, which is one of the methods of nonlinear optimization. To eliminate intersection of two surfaces we usean inequality system presented in [15, 16]:

$$\begin{cases} (\overline{p}_{пов2\_1} - \overline{p}_{пов1\_1})^T \cdot \overline{n}_1 \ge 0, \\ \dots \\ (\overline{p}_{пов2\_i} - \overline{p}_{пов1\_i})^T \cdot \overline{n}_i \ge 0; \end{cases} \tag{5}$$

where $\overline{n}_i$ is the normal vector of the $i$ point of the first mating surface.

Graphic presentation of setting of alignment problem in the model describing the best fit is shown in Fig. 2.

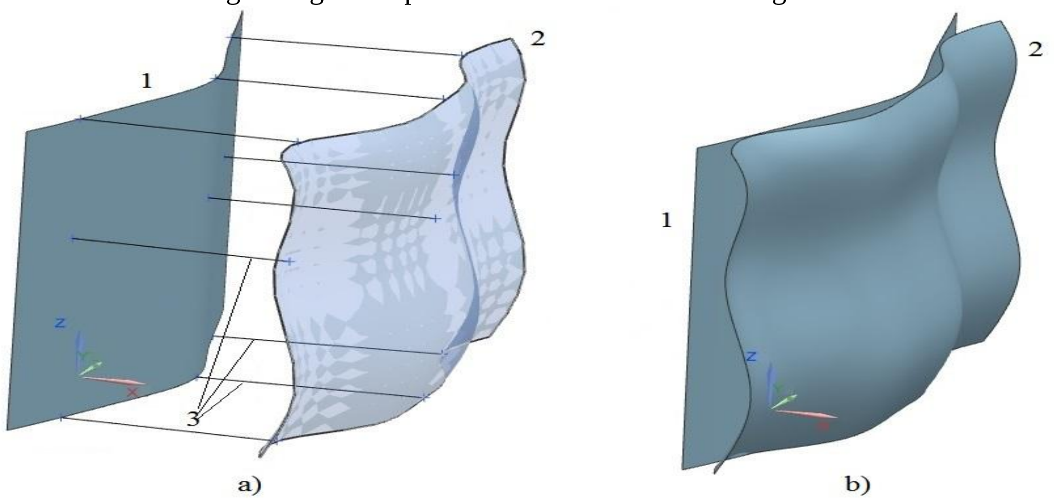


**Fig. (2).** Graphic representation of setting of alignment problem:
a) Surfaces 1 and 2 before alignment wherein 3 is normal to the surface 1 passing through the mating point;
b) Surfaces costate

The algorithm takes into account the mating conditions, which formalized by the motion of vector $\overline{D_1}$ in view of surfaces crossing terms.

### 2.3 Uncertainty estimate of the mating surfaces

Uncertainty estimate for mating surfaces is performed on the basis of multiple simulation of mating with different initial conditions according to the Monte Carlo method. As different initial conditions, such as: 1) the set of realizations of form deviations of the mating surfaces defined by the amplitude and phase; 2) the set of implementations describing the assembly and conditions determined by the direction vector of the assembly effort application may be used.

The required number $n_n$ of implementations of interfaces defined by taking into account the required accuracy $\delta$ at a given confidence level $P$. Assuming that the modeling stray fields of mating uncertainty are described by the normal distribution with the value of the standard deviation $\sigma_x$, the number of implementations $n_n$ can be determined by the formula:

$$n_n \geq (z_p \cdot \sigma_x / \delta)^2, \tag{9}$$

где $z_p$ - квантиль нормального распределения.

where $z_p$ is quintile of normal distribution.

## 3. Results

While experimental studies for improvement of the proposed method and algorithm were conducted, the interface of the two rings was simulated with nominal mating surface in the form of truncated cones. The height of rings is 40 mm, diameters of mating surfaces in the lower section are 54.72 mm and the angle at the base of the cone is 6 °.

Form deviation of rings surfaces in cross-section is taken as a constant from top to bottom. To describe the mating surfaces of the rings in each of the nine sections a grid of 80nodal points was set, by which construction of spline surface was carried out.

### 3.1. Shape deviations schemes

On the basis of the analysis results of measurement of the ground surfaces it was found that the form deviation of the surfaces has a harmonic character and can be presented as follows:

1) For a surface of a covered ring:

$$dF_1 = A_1 \cdot (1-\sin(t)/3) \cdot \cos(k_1 \cdot t + \pi/2) + A_2 \cdot (1-\cos(t)) \cdot \sin(k_2 \cdot t + \pi/2) - \Delta_1, \tag{10}$$

where $A_1$, $A_2$ are harmonic amplitudes, mm;

$k_1$, $k_2$ are harmonic frequencies;

$t$ is an angle of the conical surface section, varies in the range from 0 to 360 degrees;

$\Delta_1$ is a constant.

2) For a surface of a covering ring:

$$dF_2 = -(A_1 \cdot (1-\sin(t)/2) \cdot \cos(k_1 \cdot t + \pi/4) + A_2 \cdot (1-\cos(t)/4) \cdot \sin(k_2 \cdot t + \pi/2) + \Delta_2), \tag{11}$$

where $\Delta_2$ is a constant.

The values of the variables of the formulas (1) and (2) are shown in Table. 1.

**Table 1.**Characteristic values of form deviations of the mating surfaces.

| Parameter | $A_1$,μm | $A_2$,μm | $k_1$ | $k_2$ | $\Delta_1$,μm | $\Delta_2$,μm |
|---|---|---|---|---|---|---|
| Value | 29,4 | 14,7 | 3 | 6 | -24,4 | -38,7 |

### 3.2. The results of evaluation of the uncertainty of the mating surfaces

Research of the matings uncertainty was conducted for confidence level $P$ equal to0.999 and the quintile of the normal distribution $z_p$ was equal to 3.09. The adopted accuracy $\delta$ of the coupling characteristics was0.5 microns; $T$ tolerance for the deviation of the mating rings was 0.01 mm. The minimum required number of realizations of the random process for set-up parameters according to (5) reaches 106.

The calculation of 120 interfaces with the rotation step of the covering ring around an $z$ axis constituting 3º was conducted, that provided a change of reference condition of mating (interface) calculation. Fig. 3 shows a set of coordinates of the centers of the outer ring axis in the XOY plane while mating.

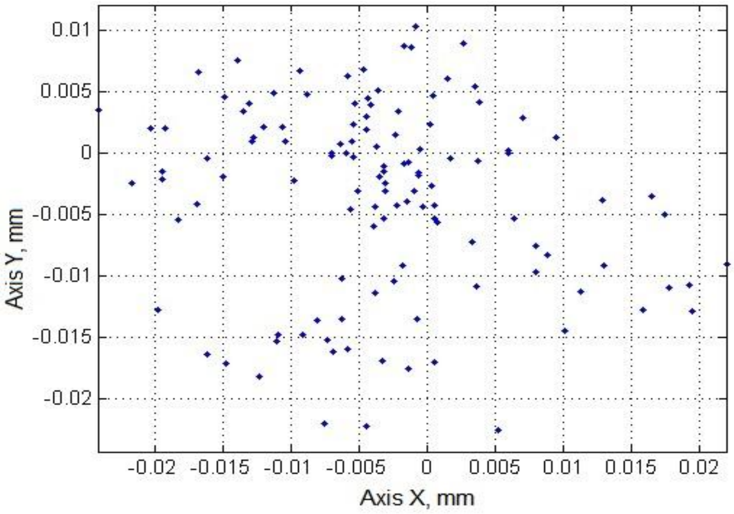


**Fig. (3).** A set of coordinates of the centers of the outer ring axis in the XOY plane while mating

For convenience in analysis the centers coordinates were presented as polar plot, in which each point is characterized by the radius vector $\rho$ and the corresponding azimuth $\varphi$ varying from 0 to 360º. The shift $\Delta z$ of the end surfaces of the outer ring relative to the inner ring along the Z axis was determined additionally for the analysis of relative position of the mating surfaces. For each implementation the error of calculation of interface $\xi_a$ was calculated, which is equal to the value of the objective function (2). Histograms of the distribution of spatial mating uncertainties are shown in Fig. 4.

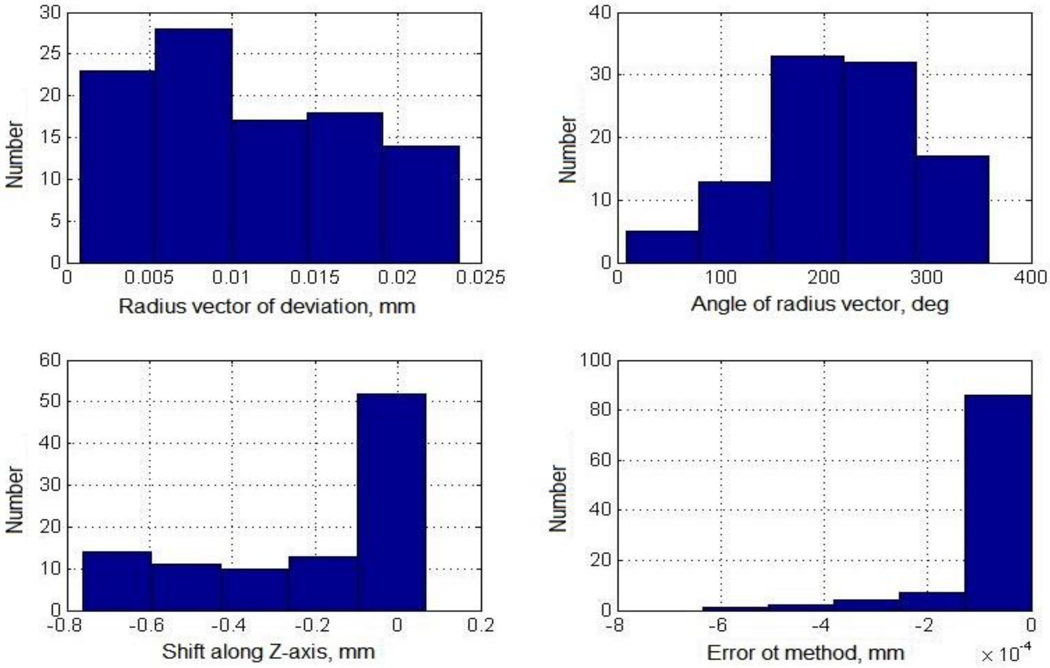


**Fig.(4).** Histograms of the distribution of spatial mating uncertainties

For each considered parameter of uncertainty, as well as for mating calculation error, the expectation was calculated, as the mean-square deviation, and upper and lower bound for the error of measurement of the considered point coordinates with a confidence level of 99.73% was calculated too. These calculations are given in Table. 2.

**Table 2.** Errors mating outer ring

| Parameter | M | SKO | MAX | МИН |
|---|---|---|---|---|
| $\rho$ , μm | 8,43 | 3,81 | 23,68 | 0,80 |
| $\varphi$ , deg. | 211,85 | 77,45 | 359,48 | 8,12 |
| $\Delta z$ , μm | -426,95 | 137,65 | 66,00 | -759,91 |
| $\xi_a$ ,μm | -0,28 | 0,11 | 0,00 | -0,63 |

Analysis of the results showed that the developed algorithm characterized by a sufficiently small value of mating calculation error, which is not more than 0.7 μm. Uncertainty of mating conical surfaces position was not more than 24 μm, which corresponds approximately to 30% of the form deviation of mating surfaces.

## Conclusion

The paper describes the estimation model of uncertainties spatial mating of high-precision parts,which are absolutely rigid bodies. The model can be used in the design of mechanical systems to estimate the effects of geometric variations of elements on the accuracy of assembly parameters. Another its application may be determination of a more acceptable version of the assembly and adjustment of high precision mechanical systems based on the results of measurements of the geometry of mating parts.

Implementation of the method is performed on an example of the assembly of conical with a form deviation in cross direction within 0.1 mm. The mating uncertainties are defined for geometrical parameters of the coordinates of the movable ring axis center. Analysis of the results showed that the developed algorithm is characterized by a sufficiently small value of mating calculation error. The uncertainty of the position of mating cone surfaces was not more than a third of the value of form deviation of mating surfaces.

The results of the research allowed the conclusion about the significant influence of the form deviation of the mating surfaces on the uncertainty of their conjugation. Form deviations of manufactured parts have a significant impact on their relative position. Using the uncertainty estimates for the optical systems of calculations to evaluate manufacturing tolerances and tolerances to solve task assignment, taking into account their impact on the physical processes in mechanical systems. This model can be extended to conjugation of other standard surfaces.

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