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Analysis of free and forced vibration of FGM rectangular floating plates (in contact with fluid) using the theory of Mindlin

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Abstract

Vibrations of relatively thick rectangular flat plates with constant thickness in contact with fluid are of particular importance to construction of power plants, airports, ships in water, etc. In this study, free and forced vibrations of plates known as FGM with four different classical supports under transverse loading were studied. In this study, for rectangular plates with two parallel edges on simply support, vibration characteristic equations to obtain the natural frequencies are obtained using the exact solution which is used for detailed analysis of forced vibration. Then, numerical results of free vibration and effect of different geometric parameters like plate length to width, plate thickness to width, fluid height, fluid density, and so on, on parameters of free vibration are analyzed. At the end, using mode shape expansion method, the equations governing the behavior of forced vibration in relatively thick rectangular plate are obtained and after validating the obtained accurate answer, different parameters on behavior of plate on behavior of plate under dynamic load are investigated.

*Keywords***:** floating rectangular plate, natural frequency, forced vibrations, dynamic response, Mindlin theory.

1-Introduction

Very large floating structures are made in two floating and semi-floating types. Semi-floating structures are kept on water surface using tubular columns or balancing structural elements. This type of structure is used for restless sea conditions with high waves. The floating types or pontoons remains on the water surface like a large plate. Floating structures are mostly suitable for calm waters such as ponds, bays and coastlines. Very large flat floating structures are briefly called VLFS. These floating structures are improved compared to the traditional methods for the following reasons: they can be built simply and quickly , they can be easily moved and expanded if necessary, water depth does not affect their overall structure, their position relative to water surface is static and thus can be used to build airports and docks, they do not have negative impact on the environment, buildings and people in them are safe from potential earthquake hazards since earthquakes energy is taken by the sea (is damped) [1] .
Early work of John [2] on displacement of on a thick and rigid plate floating was presented. Idogho in 2009 investigated design of connection to reduce Hydroelastic behavior of a floating structure by connecting beams together. They found that semi-rigid connections are very effective in reducing Hydroelastic behavior [3]. The first application of very large floating structures has been reported as Floating boat bridges [4].

Very large floating structures can be used for fuel storage. For this purpose flat tankers are built as box -shaped that is placed one after another side by side on the surface of sea. Then, they are each other and also to the sea floor using some equipment. Since the fuels are highly flammable, with storing those in these reservoirs, dangers caused by their explosion and flame to living environment are prevented. Japan country has two floating oil storage reservoir, one of them is in Shirashima with capacity of 5.6 billion liters and the other in Kamigoto city with capacity of 4.4 billion liters [5]. One of possible applications of VLFS in the not too distant future is construction of wind and solar power to generate electricity.In 1979, Bangladesh purchased a floating plant from Japan [6]. One of the interesting applications of VLFS is building floating airports. Researchers have conducted extensive studies on possibility of building floating airports on seas and near coasts. Air traffic has also increased with the growth of cities and the need to expand aerial airports is felt more. In Asia, especially Japan country, there has been significant growth in the construction of floating airports. Kansai International Airport, which is located in Osaka, Japan, is the world's first airport built completely at sea, despite being located on an artificial island [7].
Figure 1 shows a view of a floating airport in Tokyo, Japan, and Figure 2 shows a view of the proposed plan to build an airport near the city of Tokyo, Japan.

Figure 1. Floating airport, Tokyo Japan

Figure 2. Proposed plan for construction of a floating airport

1-1-Functionally graded materials (FGM)

According to definition, functionally graded materials are materials used for creating gradual changes in characteristics of microstructures components or compositions. In [8, 9] with three-dimensional theory of elasticity, buckling and free vibration of a number of plates were solved by exact method. Also in [10] stress and free vibration of composite asymmetric layer plates with angle-ply and cross-ply layers were investigated by three-dimensional elasticity exact method. With the increasingly growth and of technology and complexity of and properties of materials used in plate, possibility of exact solution of their elasticity theory also correspondingly decreased. In this regard, scientists used two-dimensional theories including classical theory of plate, first order shear theory and higher-order shear theory to exact solution of plate's free vibration under simple boundary conditions [11]. In [12-14] using the classical theory of plate, free vibrations of asymmetric layered plates were studied and exact method of free vibration frequency of layered asymmetric angle-ply and cross-ply plates were obtained. Asemi et al. [15] proposed post-buckling and non-linear bending analysis of FGM annular sector plates based on three dimensional theory of elasticity in conjunction with non-linear Green strain tensor. Their results indicated that due to the stretching–bending coupling effects of the FGMs, the post-buckling behavior of movable simply supported FGM plates is not of the bifurcation-type buckling.

2-Materials Theory of FGM and Mindlin plate

2-1-Theory of plate analysis

In general, each of the displacement components on plate is a function of location and time. However, given that stress and strain components are derived from the displacement, they are also a function of location and time. But in order to obtain the governing equations, displacement dependency to time and place is usually used. Generally displacement of different points across the page is as follows:

Displacement indirection 1 (x)$U_1(x_1,x_2,x_3,t)$
Displacement in direction 2 (y) $U_2(x_1,x_2,x_3,t)$
Displacement in direction 3(z) $U_3(x_1,x_2,x_3,t)$

Existing theories to analyze the pages are divided into three major categories:

1) Classical plate theory(CPT)
2) Mindlin or first-order shear deformation plate theory(FSDT)
3) Higher-order shear deformation plate theory(HSDT)

2-2-Mindlin theory and equations governing behavior of plate

First, the assumptions based on which this theory is built are brought. Then, fields of stress, strain, displacement and equations dominating vibrations of thick plates are obtained. In Mindlin theory of thick plates, to study the vibrations caused by small displacements ,a flat isotropic relatively thick rectangular plate , with uniform thickness h, length of a, width b, modulus of elasticity E, Poisson's ratio v, shear modulus G = E/ (2 (1 + v)) and density per volume unit ρ, Cartesian coordinate system with components x_1, x_2 and x_3 that according to Figure 3 is based on the middle layer of plate , are considered

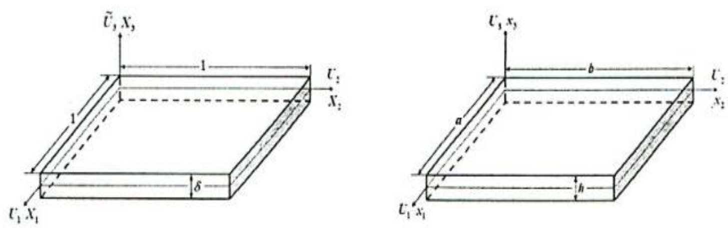

Figure 3. Relatively thick rectangular plate with coordinate system intended in the middle layer

Plate displacement along the axes $x_1$and x_2 as shown in Figure 3 are shown by U_1andU_2. And displacement of plate along the perpendicular to the middle layer of plate is displayed by U_3. in order to obtain more accurate responses to relatively thick plates) ((0.05 <h) /a <0.2 compared to the classical theory of thin plates, Mindlin–Reissner considered assumptions to make behavior of the plate against static and dynamic loads assumptions closer to reality, these assumptions include :

1) The middle plane of plate (x_3 = 0) lacks displacement along $x_1$andx_2. Therefore, $U_1$and U_2displacements should be provided as follows.

$$U_1 = f_1(x_3)\psi_1(x_1,x_2,t)$$
$$U_2 = f_2(x_3)\psi_2(x_1,x_2,t) \qquad (1)$$

So that

$$f_1(x_3)_{x_3=0} = f_2(x_3)_{x_3=0} = 0 \qquad (2)$$

2) The initial flat sections perpendicular to the center line of the plate before deformation continue to remain flat after deformation (Fig. 4). It is observed that for section's remaining flat after deformation of plate U_1 *and* U_2 displacements should be linear function ofx_3. In this case, not only $\frac{\partial U_1}{\partial x_3}$ and $\frac{\partial U_2}{\partial x_3}$ will be independent of x_3, but also assumption 1 in relation to displacement of U_1 and U_2 and middle plate are also satisfied. Also, the slope of the layers that make up the plate for each assumed section with certain values must be constant across the plate after deformation, so we can write:

$$U_1 = -x_3\psi_1(x_1, x_2, t), \quad (3)$$
$$U_2 = -x_3\psi_2(x_1, x_2, t), \quad (4)$$
$$U_3 = \psi_3(x_1, x_2, t), \quad (5)$$

In the above relations t represents time, $\psi_1$and $\psi_2$slope due to Flexural respectively along the axes x_1, x_2 and ψ_3, transverse displacement along axisx_3. Assuming strain- linear displacement, strain field for Mindlin plates in Cartesian coordinate system will be as follows [16-19].

$$\varepsilon_{11} = \frac{\partial U_1}{\partial x_1} = -x_3\psi_{1,1}, \quad (6)$$
$$\varepsilon_{22} = \frac{\partial U_2}{\partial x_2} = -x_3\psi_{2,2}, \quad (7)$$

ε (8)

ε (9)

ε (10)

ε (11)

According to Figure 4 and by using the equations (10) and (11) it can be seen that if $\psi_1 = \psi_{3,1}$ *and* $\psi_2 = \psi_{3,2}$, shear strains $\varepsilon_{13}$and ε_{23} are zero, and in this case we have $\left|\frac{\partial U_1}{\partial x_3}\right| = \left|\frac{\partial U_3}{\partial x_1}\right|$ *and* $\left|\frac{\partial U_2}{\partial x_3}\right| = \left|\frac{\partial U_3}{\partial x_2}\right|$ or in other words after deformation, flat sections not only remain flat but also are perpendicular on intermediate page and this assumption is also true in classical theory of thin plates or theory of Kirchhoff plates, so in Mindlin theory of relatively thick plate, effects of non-zero shear strains $\varepsilon_{13}$and ε_{23} *are* taken into account and since in this theory shear strains $\varepsilon_{13}$and $\varepsilon_{23}$are non-zero according to the relations (10) and (11) are independent of x_3, therefore, indicate availability of study non-zero strains in free surface of $x_3 = \pm h/2$plates; that in fact these surfaces should be zero. To compensate for this error, shear correction factor κ is suggested that in Reissner theory of plates, $\kappa^2 = 5/6$ and in Mindlin theory of plate $\kappa^2 = \pi^2/12$[18].

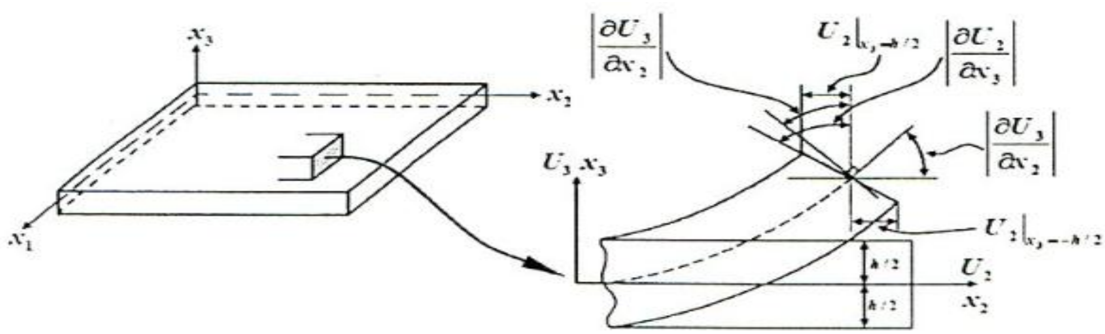

Figure 4. Deformation of cross-sectional of plate with first order shear theory (Mindlin)

3- Methods and steps of research

3-1- Free vibrations of FGM plate in contact with fluid using Mindlin theory

Geometry and coordinate system for a rectangular plate in contact with the restricted fluid (plate floating on the water) is shown as Figure 5.

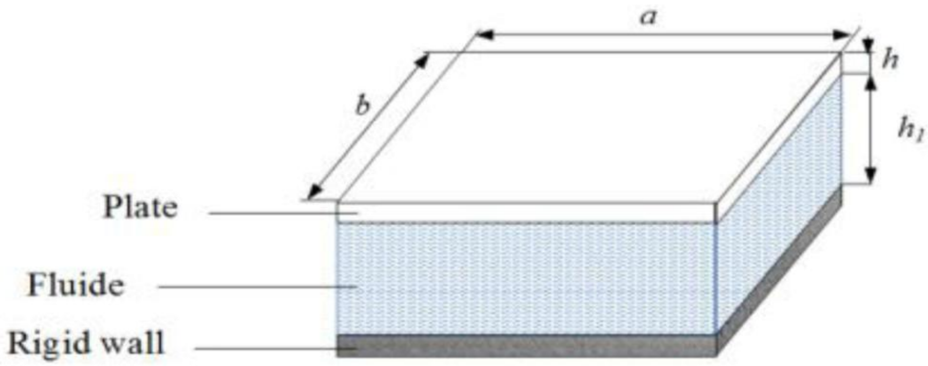

Figure 5. Model of plate floating on a fluid restricted by a rigid wall

3-1-1 Determiningμ_f

μ_f is the number of pure wave that in vibratory motion scope in directions $x_1$and x_2, we have $\mu_f = \sqrt{{\mu_1}^2 + {\mu_2}^2}$. Korbua et al used simple form of $\mu_f = \frac{\pi}{2}\sqrt{1+\eta^2}$for all boundary conditions . Although value of μ_f for different boundary conditions varies, they did not consider the effect of boundary conditions onμ_f. In addition, the wave number presented in reference is independent of air frequency and is constant during change of mode number. In the present work, wave number used in reference has been modified using Mindlin parameters. It is noteworthy that a higher degree of edges restrictions (from simple to fixed support) are applied to two edges of the rectangular plate. Parameters of following wave number when at least one of the edges of the rectangular plate vibrates freely have been introduced (in the boundary conditions SSSF, SFSF, SCSF).

$$\mu_1 = \sqrt{\alpha^2 - \left(\frac{n\pi}{a}\right)^2}\ \mu_2 = \frac{2\pi}{L} \tag{12}$$

In the above relation we have:

$$\alpha^2 = \frac{{\beta_a}^2}{2a^2}\left[\frac{\delta^2}{12}\left(\frac{1}{\kappa^2 v_1} + 1\right) + \sqrt{\left(\frac{\delta^2}{12}\right)^2\left(\frac{1}{\kappa^2 v_1} - 1\right)^2 + \frac{4}{{\beta_a}^2}}\right] \tag{13}$$

Where, L is width of tanks and β_a is parameter of dimensionless frequency in vacuum.

For three other boundaries conditions (SSSS, SCSS, and SCSC), $\mu_1$does not change, while $\mu_2$has changed with the following equation:

$$\mu_2 = \frac{m\pi}{a} \tag{14}$$

In Figure 6 natural frequencies for different boundary conditions are shown using numerical solution.

With the increase of geometric constraint in edges, the natural frequency increases too. Based on these results we can say that the lowest and highest natural frequency is respectively for rectangular plates with boundary conditions S-F-S-F and S-C-S-C is. Therefore, with increase of constraint on the edges, flexural rigidity of plate is increased, which raises the natural frequency.

3-1-2Finite element model

To ensure the results obtained from free vibration analysis using Mindlin theory, a rectangular plate of FGM was modeled in ANSYS software and modal analysis was done on it. The fluid used is water that has a density of $1000\ \mathrm{Kg.m^{-3}}$ and bulk moduli is 2.15GPa. Also, rectangular plate was placed inside a cube tank. Sample model of relatively thick rectangular plate in contact with fluid is shown in Figures 7 to12.

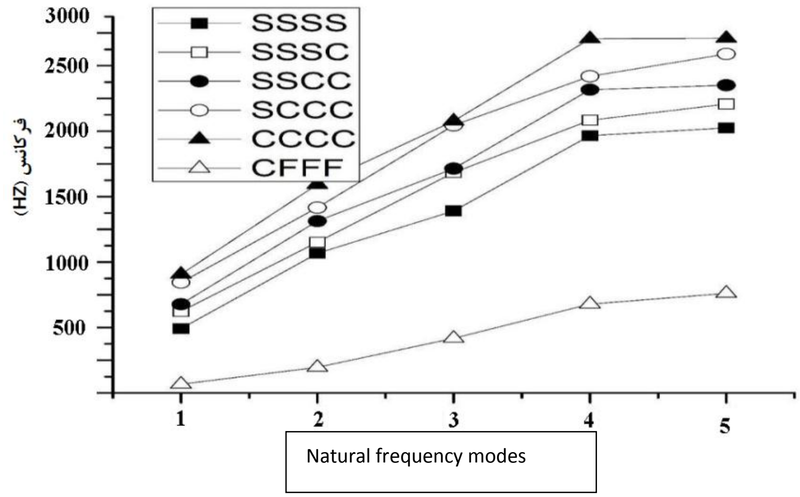

Figure 6. Frequency modes for different boundary conditions using numerical solution

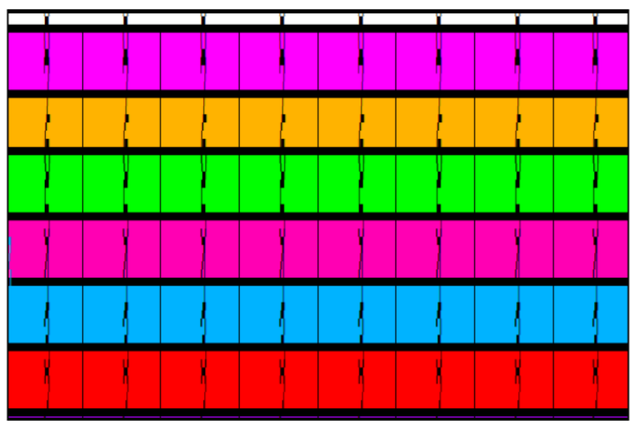

Figure 7. Square page with 8 layers

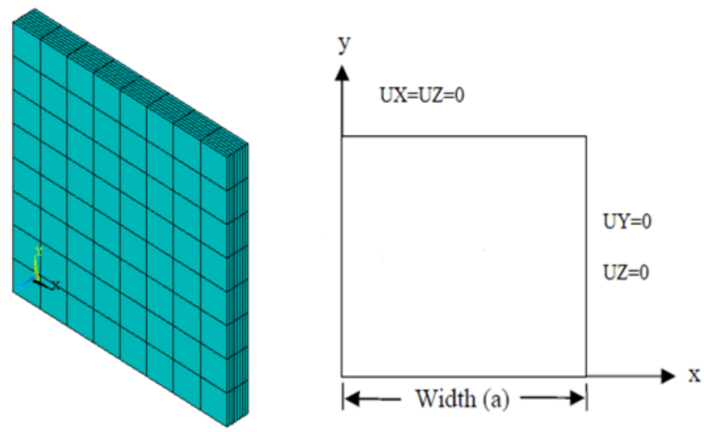

Figure 8. FGM Page with 8 layers with of different materials

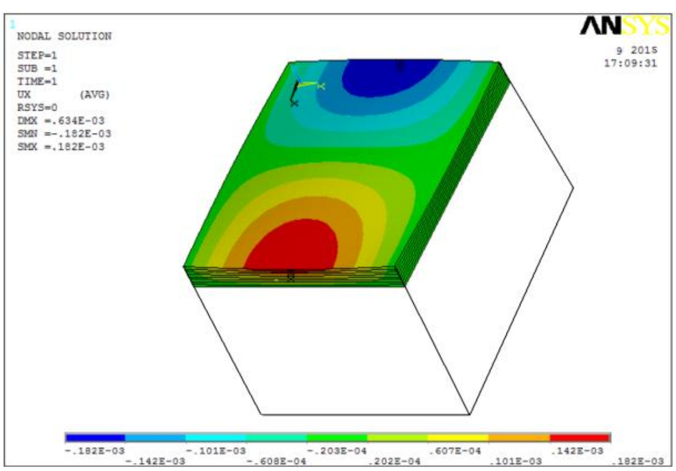

Figure 9. First natural frequency of FGM rectangular plate in contact with the fluid used in ANSY software

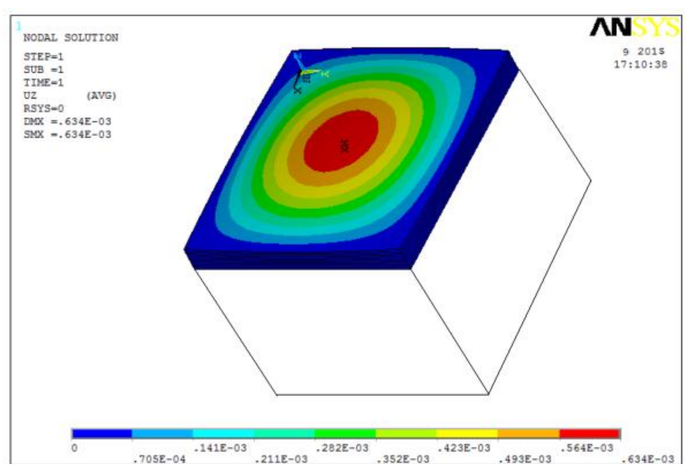

Figure 10. second natural frequency of FGM rectangular plate in contact with the fluid used in ANSY software

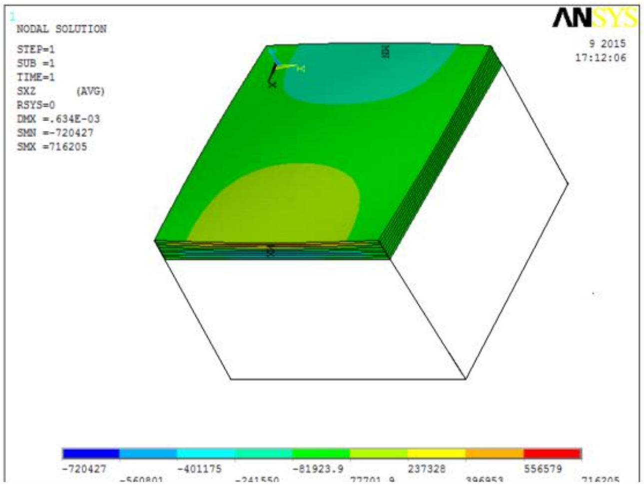

Figure 11. Third natural frequency of FGM rectangular plate in contact with the fluid used in ANSYS software

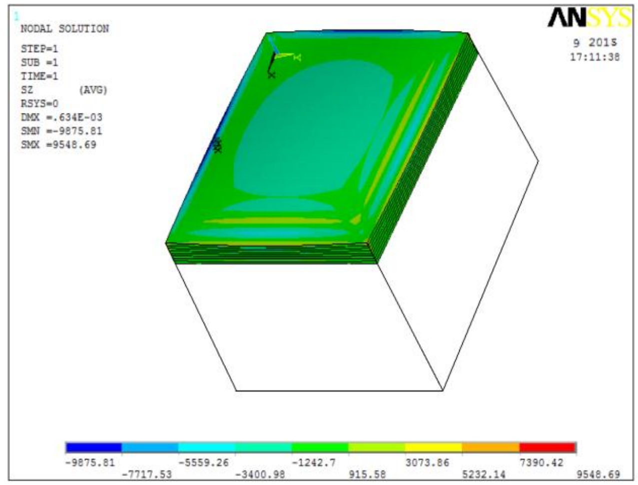

Figure 12. Fourth natural frequency of FGM rectangular plate in contact with the fluid used in ANSYS software

3-2 Forced vibrations of FGM plate in contact with the fluid

Geometry and coordinate system for a rectangular plate in contact with restricted fluid (plate floating on the water).

In this section, response of system will be considered for two types of time input:

A) Stepwise input :

B) F (t) = 1000 [N]; t > 0

C) B) harmonic input of excitation bat frequency of 100 Hz:

D) F) t) = 1000: sin (100 t) [N]; t> 0

Dynamic balance relations in absence of inside page forces and elastic foundation coefficients for rectangular plate with assumptions of first order shear theory of Mindlin in three directions of Cartesian coordinate system by substituting linear stress-strain relations, and strain - displacement relations for FGM are expressed as follows:

$$\frac{D}{2}[(1-\nu)\nabla^2\psi_1+(1+\nu)(\psi_{1,11}+\psi_{2,21})]$$
$$-\kappa^2 Gh(\psi_1-\psi_{3,1})=\frac{1}{12}\rho h^3\ddot{\psi}_1 \quad (15)$$
$$\frac{D}{2}[(1-\nu)\nabla^2\psi_2+(1+\nu)(\psi_{1,12}+\psi_{2,22})]$$
$$-\kappa^2 Gh(\psi_2-\psi_{3,2})=\frac{1}{12}\rho h^3\ddot{\psi}_2 \quad (16)$$
$$\kappa^2 Gh[\nabla^2\psi_3-(\psi_{1,1}+\psi_{2,2})]-p\left(x_1,x_2,t\right)$$
$$=\rho h\ddot{\psi}_3$$

In the above equations$D=\frac{eh^3}{12(1-\vartheta^2)}$ *is* flexural rigidity coefficient, ψ_1 and ψ_2 are respectively slope in the direction of x_1 and x_2,$\psi_1$is displacement in direction of x_3, is shear correction factor, G is shear modulus and p is forced force.

3-2-1- Finite element model

To ensure the results obtained from analysis of free vibrations using Mindlin theory, a rectangular plate of FGM was modeled in ANSYS software and modal analysis was done on it. The model used in this section is the same model used in free vibration part. Because in ANSYS software for performing forced vibration analysis, it is required do modal analysis (vibration); then forced vibration analysis is done with considering vibrational forces.After analysis, according to Figure 13, stepwise and harmonic vibrational forces are entered.

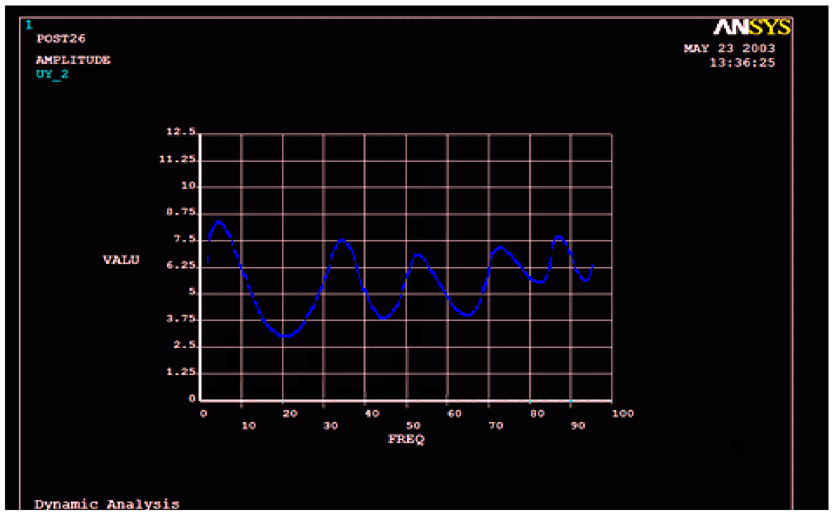

Figure 13. Variations of forced vibration response of FGM rectangular plate

5. Results and discussion

5-1.Impact of thickness increase on free vibration of FGM plate in finite element model

As shown in Figures 14 and 15, increase of thickness cause increase of all natural frequencies of the object. Of course, this trend is normal because increased thickness improves hardness of the object. Increased hardness rises amount of natural frequencies, also reduces amount of deformation.

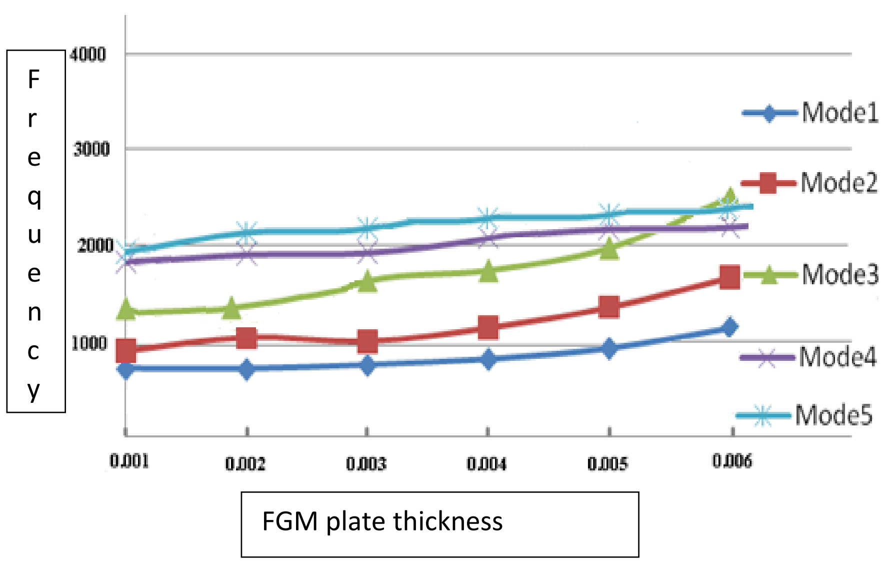

Figure 14. Impact of thickness change on natural frequencies of FGM plate

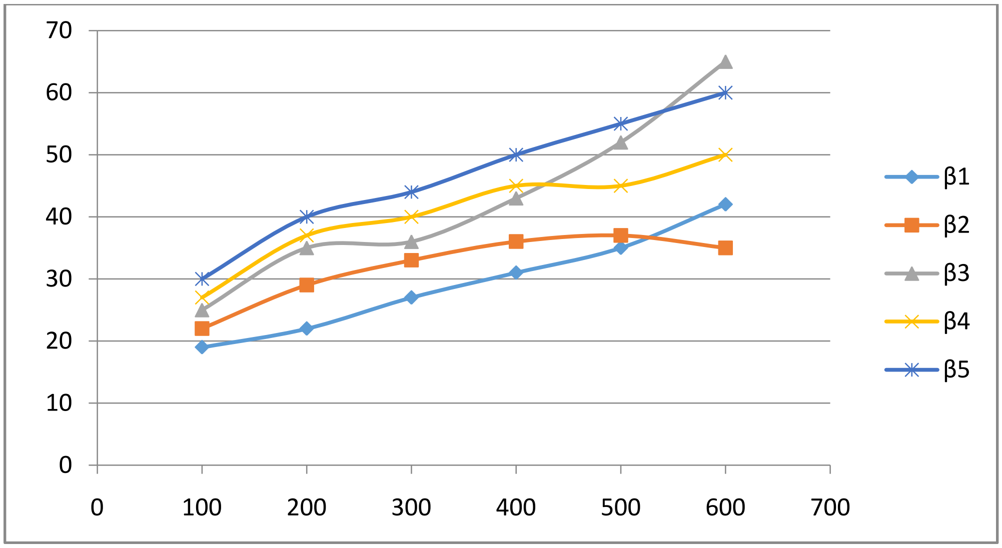

Figure 15- Impact of thickness change on the frequency parameter $\beta = \omega a^2 \sqrt{\frac{\rho h}{D}}$

With increase of length to thickness ratio in FGM plate as shown in Figures16 and 17, natural frequencies do not change substantially. Thus, increasing ratio of length to thickness in FGM plates due to impact on other parameters such as mass will not have great impact on vibration

5-2- Impact of fluid density on free vibration FGM plate in finite element model

In this study, fluid density or liquid, due to taking water into account, was considered as liquid in contact with the fluid. Thus, in entire the study fluid density is considered as 1000. But in order to investigate the effect of density in finite element software, fluid density is considered as 800, 900, 1000, 1100, 1200 and 1300.

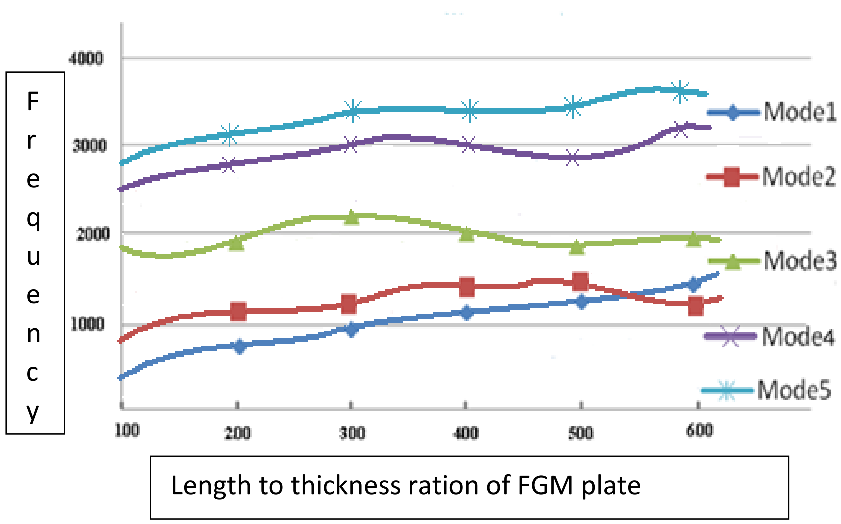

Figure 16-Impactof change of length to thickness ratio on natural frequencies of FGM plate

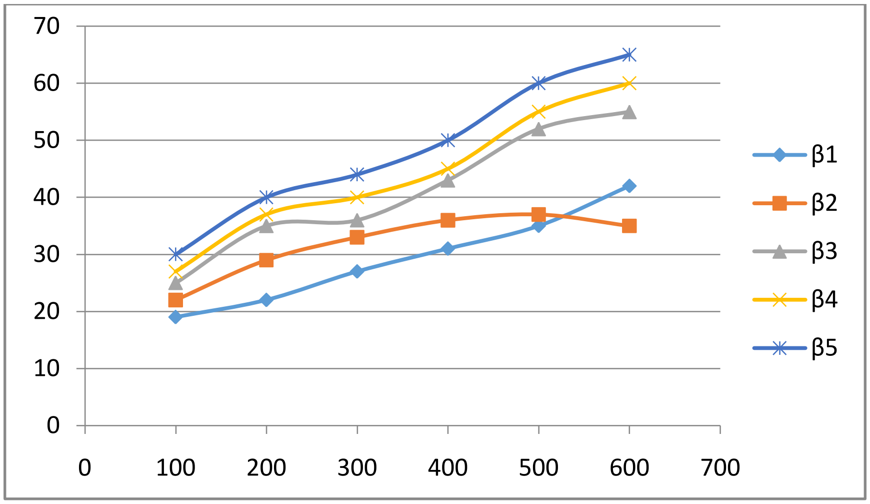

Figure 17- Impact of change of length to thickness ratio on the frequency parameter $\beta = \omega a^2 \sqrt{\frac{\rho h}{D}}$

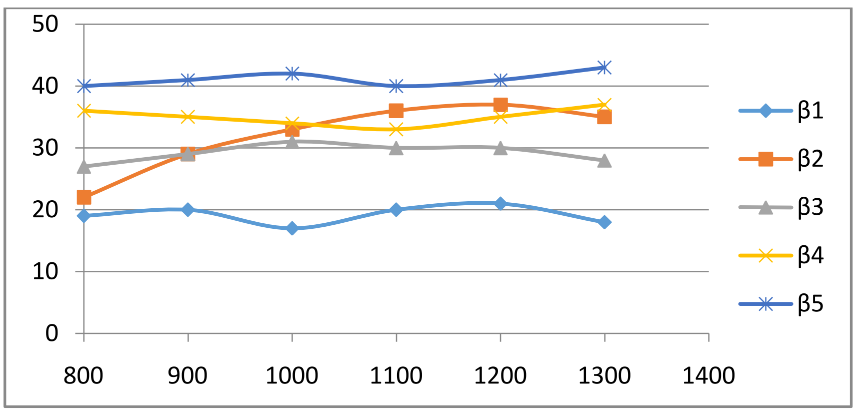

Figure 18- Impact of density change on frequency parameter $\beta = \omega a^2 \sqrt{\frac{\rho h}{D}}$

As shown in Figure 18for the boundary conditions C-C-C-C, shown increase of density does not have any effect on natural frequency and dimensionless parameters of frequency. Because fluid in contact with plates, acts like a damper, that is fluid only has damping effect and reduces vibration amplitude.

5-3-Impact of changes in the parameters on Forced Vibration of FGM Plates

5-3-1-Thickness of FGM plate:

One of the most important parameters in the design of industrial panels is their thickness. Because the thickness affects the cost of these pieces on one hand, and increases the piece weight on the other hand. Given that these pieces are often movable; their weight increase will raise their energy consumption. Figures 19 to 21 show impact of thickness on vibrations and dynamic responses.

As noted in above figures, increase in thickness of FGM plates directly affects vibrations of these plates.

5 -3 -2- Effect of fluid height on transverse displacement of FGM plate:

As noted in previous sections, fluid in contact with plate acts like damper. In Figures 22 to 24 effect of fluid height change for heights of 0.01, 0.015 and 0.02 are shown.

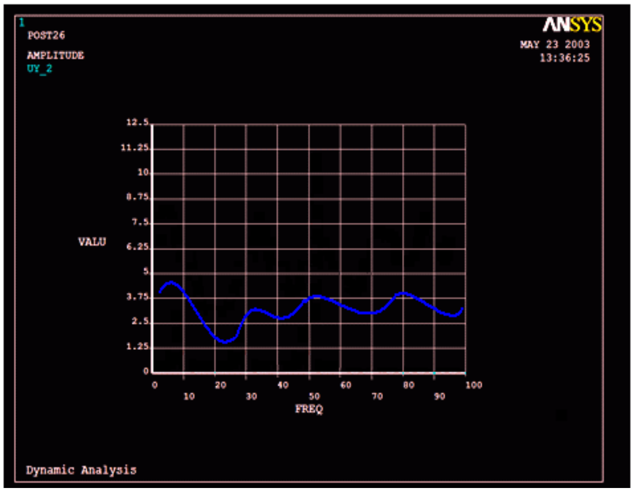

Figure 19. changes in forced vibration of rectangular FGM plate in ANSYS software for thickness 0.1 mm

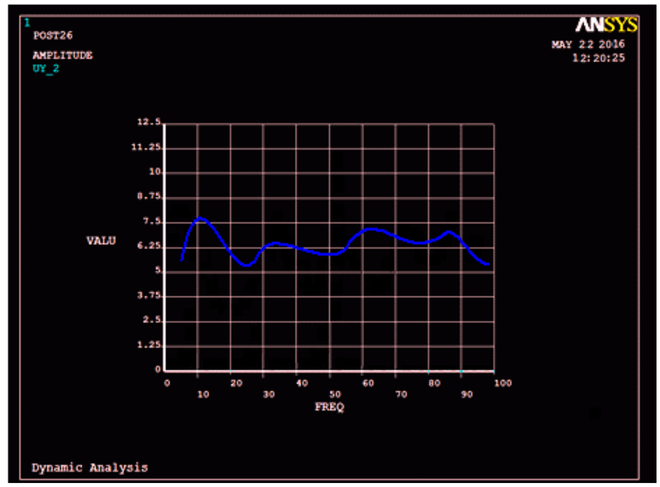

Figure 20. changes in forced vibration of rectangular FGM plate in ANSYS software for thickness 0.15 mm

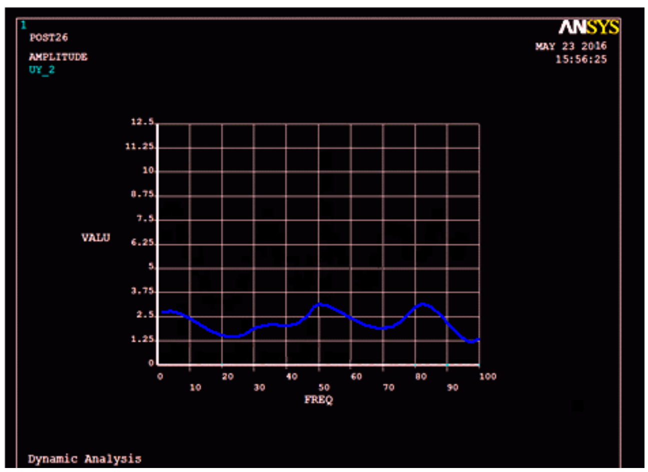

Figure 21.Changes in forced vibration of rectangular FGM plate in ANSYS software for 0.2 m

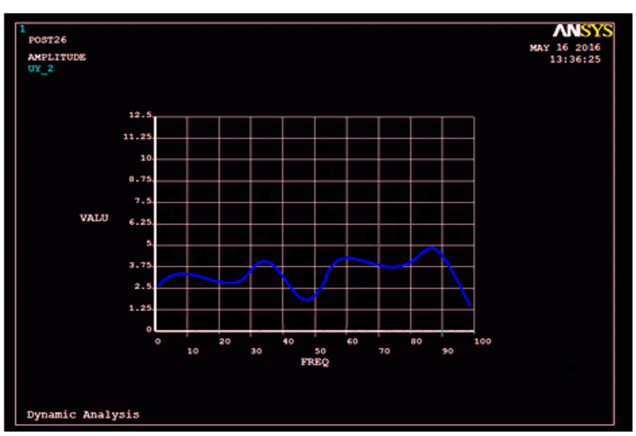

Figure 22.changes in response of forced vibration of rectangular FGM plate in ANSYS software for fluid with height 0.01 mm

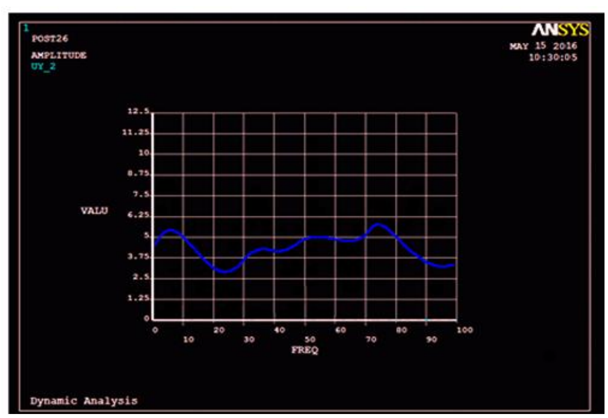

Figure 23. changes in response of forced vibration of rectangular FGM plate in ANSYS software for fluid with height 0.015 mm

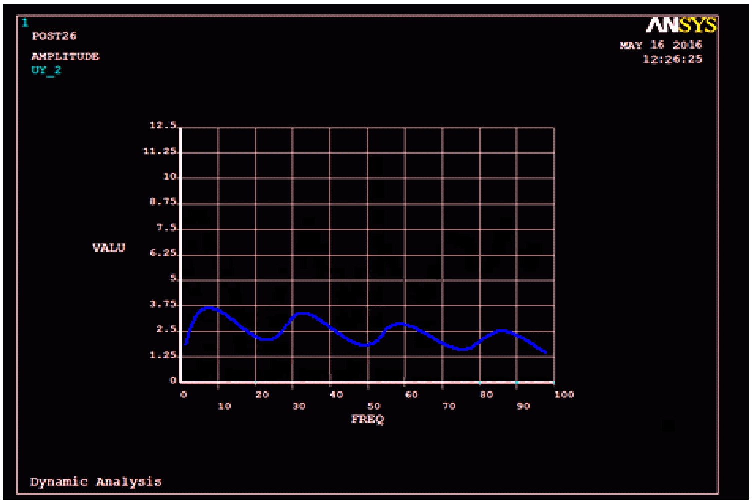

Fig 24. Changes in response of forced vibration of rectangular FGM plate in ANSYS software for fluid with height 0.2 mm

As can be seen in Figures 22 to 24, increase of fluid height can decrease vibration amplitude. This effect is caused by increased fluid damping properties due to the height of the fluid.

5-3-3-Effect of fluid density on transverse displacement of plate FGM:

Fluid density does not have effect on natural frequency and only increases system damping. Increasing damping, as shown above, causes decrease of vibration amplitudes. Thus, increasing fluid density reduces its vibration amplitude too.

5-3-4-Effect of length to width on transverse displacement of FGM plate:

According to Figures 25 to 27, one of important parameters in the design of industrial panel is their ratio of length to width. According to the results presented in these figures, it is observed that by increasing the length to width ratio plate, transverse displacement values are reduced and also it is observed that by increasing length to width ratio, transverse displacement fluctuation is increased.

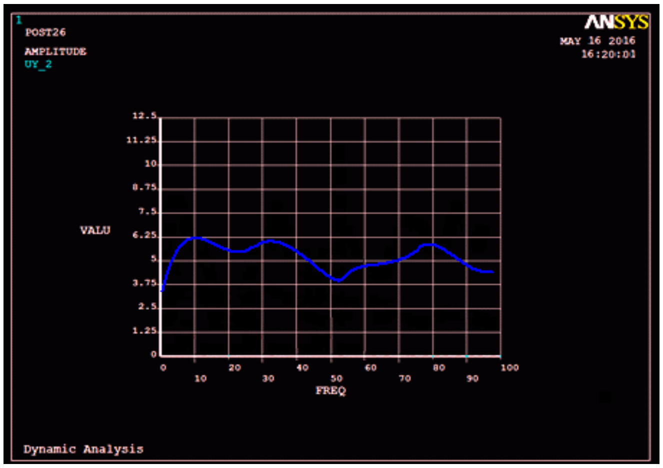

Figure 25.changes in response of forced vibration of rectangular FGM plate in ANSYS software for length to width ratio of 1

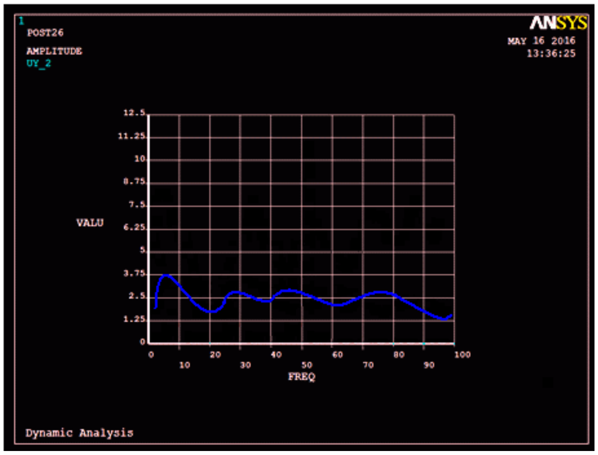

Figure 26. changes in response of forced vibration of rectangular FGM plate in ANSYS software for length to width ratio of 1.5

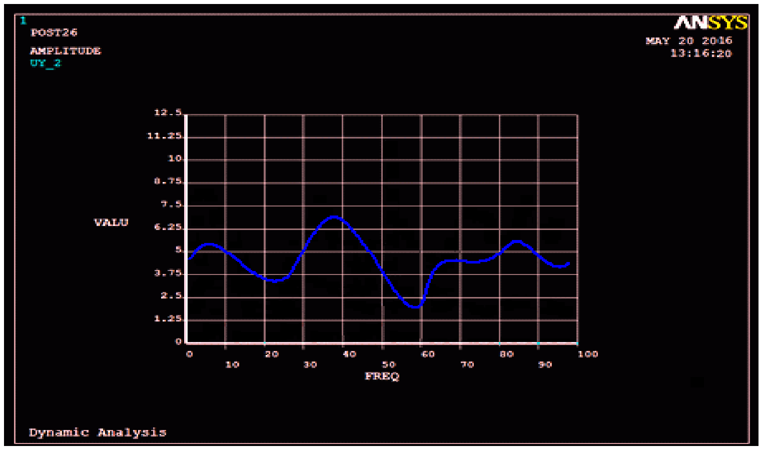

Fig 27. Changes in response of forced vibration of rectangular FGM plate in ANSYS software for length to width ratio of 2

5-4-Validationof numerical solution and finite element model results

To ensure the results obtained from the free vibration analysis using Mindlin theory, a rectangular plate of FGM in ANSYS software was modeled and modal analysis was done on it. And the results of this comparison in Tables 1 to 3, first five frequency parameter of Mindlin plate for three different boundary conditions provided by theory, are compared with finite element results. As can be seen, there is suitable consistency between current results and results of finite element software. In general, the error is the difference between the exact solution and approximate solution that is obtained by finite element method. Therefore, for calculation of error it is enough to obtain the difference between numerical solution and finite element and for obtaining percentage of error its value should be divided by value of numerical solution.

Table 1- Comparison of five first frequency parameter of FGM rectangular plate in contact with fluid in boundary conditions S-S-S-S

Natural freq.					method
5th	4th	3th	second	first	
170.524	148.412	110.342	64.512	54.345	mandolin
171.426	146.217	109.014	63.683	53.9870	Ansys
0.5	1.5	1.2	1.3	0.65	Difference

Table 2-Comparison of five first frequency parameter of FGM rectangular plate in contact with fluid in boundary conditions S-C-S-C

Natural freq.					method
5th	4th	3th	second	first	
227.356	211.648	131.142	93.412	89.641	Mindlin
223.041	209.045	129.865	92.321	88.86	Ansys
1.9	1.2	0.9	1.1	0.9	Difference

Table 3-Comparison of five first frequency parameter of FGM rectangular plate in contact with fluid in boundary conditions S-F-S-F

Natural freq.					method
5th	4th	3th	second	first	
89.281	75.895	29.452	25.895	10.246	Mindlin
90.758	74.869	28.313	25.768	10.142	Ansys
1.6	1.2	4	0.4	1	Difference

Conclusion

As shown in the above tables, difference of results obtained from analysis in ANSYS software with the results of numerical solution using Mindlin method is less than 2%, this error is an acceptable value in engineering problems.Recognizing factors affecting the vibration behavior such as geometric parameters and external factor cause that unwanted behaviors of structures can be controlled. In this study, the effect of FGM plate thickness on vibration behavior of FGM plate, especially vibration frequency in finite element model created in ANSYS software was discussed. As it was observed, increase of thickness raises all the natural frequencies of object. Of course, this trend is normal because increased thickness raises the hardness of object. Increase of hardness cause increase of natural frequencies values, also reduces amount of deformation. By increasing the ratio of FGM plates length to its thickness, natural frequencies do not change substantially. Thus, increasing the ratio of the length to thickness in FGM plates due to impact on other parameters like mass will not have much impact on vibration.

In this study, for investigating forced vibration in time domain, system response to stepwise and sinusoidal loads with different time distribution are obtained based on the extracted relations. According to the results obtained in the free vibration part, coefficients of volume ratio power and plate thickness on response of forced vibration were dealt with. And in curves, displacement of the central point in rectangular FGM plate floating on water with stepwise and sinusoidal force across the surface of the plate were examined. According to the results presented, it is observed that by increasing the ratio of length to width in plate, transverse displacement values are reduced. And also it was observed that by increasing the ratio of length to width, transverse displacements fluctuations are increased. Also increase of thickness reduces fluctuation and reduces fluctuation period.

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