J. Mater. Environ. Sci. 7 (8) (2016) 2947-2954 *Karbasi*
ISSN : 2028-2508
CODEN: JMESC

Estimation of classical hydraulic jump length using teaching–learning based optimization algorithm

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Received 09 Dec 2015, Revised 11 Feb 2016, Accepted 25 Feb 2016
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Abstract
The hydraulic jump is an interesting phenomenon in open channel flow that has been widely used for energy dissipation in hydraulic structures. The aim of this study is to investigate the applicability of teaching–learning based optimization (TLBO) algorithm for the first time in modeling hydraulic jump length over a smooth horizontal bed. Experimental data were selected from USBR reports and published literature. TLBO algorithm applied to four different regression forms: linear, quadratic, power and exponential. The TLBO method with quadratic function from among all models yielded better prediction with RMSE=0.164 m and R^2=0.974. Comparison of developed model and existing empirical equations showed that, TLBO based models have higher accuracy in prediction of hydraulic jump length over a smooth horizontal bed. Therefore, the employment of the TLBO algorithm at hydraulic engineering problems recommended for future studies.

Key words: Classical hydraulic jump, Jump Length, TLBO algorithm, Stilling basin, optimization

1-Introduction
Hydraulic jump is a phenomenon caused by change in flow regime from supercritical to subcritical with high energy dissipation and rise in depth of flow.This phenomenon increases the flow depth in a short distance and increases the turbulence and consequently causes significant energy losses. Hydraulic jump have been broadly studied because of their frequent occurrence in nature and have been extensively used as energy dissipater for hydraulic structures. Classical hydraulic jump with a smooth bed has been studied broadly and it can be shown that the ratio of the sequent depths for a classical hydraulic jump is given by the well-known Belanger momentum equation:

$$\frac{y_2}{y_1} = \frac{1}{2}(-1 + \sqrt{1 + 8{Fr_1}^2}) \quad (1)$$

Where $\frac{y_2}{y_1}$ is the ratio of sequent depths and Fr_1 is supercritical Froude number. Hydraulic jump length is one of the most important parameters in designing of the stilling basins. Obviously economical design of a stilling basin needs accurate estimation of hydraulic jump length. The length of the hydraulic jumps cannot be determined from theoretical approaches alone. Due to difficulty of locating of hydraulic jump end section (because of waves and turbulence) definition of hydraulic jump length in an actual experiment is hard[1]. Many researchers [2, 3] define the jump length as distance between the toe of jump and surface stagnation point (Figure 1). Measurement and observation of this length is relatively easier.
Safranez [4] conducted the first systematic study on length of the hydraulic jump. His Froude number range was$Fr_1 < 2.93$. Many experimental researches were conducted in the subject of hydraulic jump characteristics and hydraulic jump length [2, 3, 5, 6].

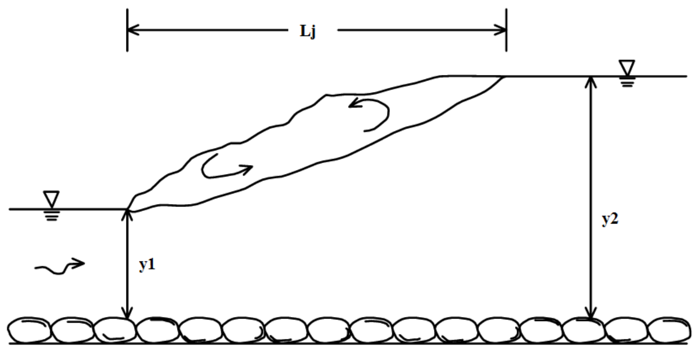

Figure 1: Sketch of hydraulic jump length over a horizontal bed [1]

Silvester [7] using regression analysis obtained the following empirical equation for jump length prediction:

$$\frac{L_j}{y_1} = 9.75(Fr_1 - 1)^{1.01} \quad (2)$$

Where L_j is jump length, Fr_1 is Froude number upstream of jump and y_1 is flow depth before jump.Mohamed Ali [8] studied the effect of cubic roughness on hydraulic jump length over it. He concluded that cube roughness can clearly reduce the hydraulic jump length. Carollo, Ferro [1] obtained the following equation for classical and B-jump length over smooth bed.

$$\frac{L_j}{H_L} = [7.965 + 20.72(tan\alpha)^{0.39}]\left(\frac{H_L}{H_1}\right)^{-0.534} exp\left(-\frac{H_L}{H_1}\frac{1}{0.168}\right) - \left[1 - exp\left(-\frac{H_L}{H_1}\frac{1}{0.168}\right)\right] 4.124ln\left(\frac{H_L}{H_1}\right) \quad (3)$$

Where L_j is jump length, H_L is head loss across the jump, H_1 is total head at jump toe section and α is angle of upstream sloping bed relative to horizontal. Gupta, Mehta [9] using experimental data developed a new empirical equation for hydraulic jump length in a horizontal prismatic channel:

$$\frac{L_j}{y_1} = 4769.1\left[\frac{Fr_1^{2.1}}{Re_1}\right] + 25.064 \quad (4)$$

Where Fr_1 is Froude number upstream of jump, Re_1 is Reynolds number upstream of jump and y_1 is flow depth before jump.

Recently, the interest in the use of soft computing techniques such as artificial neural networks (ANNs), adaptive neuro-fuzzy inference system (ANFIS), genetic programming (GP) and decision trees (DT), has become common in hydraulic engineering problems [10-16]. Naseri and Othman [17] used artificial neural networks (ANN) to predict the hydraulic jump length over a horizontal smooth bed. Their results showed that ANN can predict the jump length with high accuracy. Comparison of empirical equations with ANN showed that ANN has higher accuracy.

TLBO algorithm is a new optimization technique and has been already employed on a large number of constrained and unconstrained benchmark problems in different engineering fields, namely thermal engineering [18, 19], computer engineering [20], environmental engineering [21], electrical engineering [22, 23], civil engineering [24-26], mechanical engineering [27, 28], and energy [29].This algorithm shows its excellent ability to find optimum solution. TLBO algorithm proved to be better than other advanced meta-heuristic optimization techniques like particle swarm optimization (PSO) and differential evolution [30]. More application of TLBO algorithm applications can be found in Rao [31].

The purpose of the present study is to develop new equations being quadratic, exponential, linear, and power functions to predict hydraulic jump length over a smooth horizontal by using TLBO method.

2-Materials and Methods

2-1-Theoretical background

Hydraulic jump length over a smooth horizontal bed is dependent on fluid properties and hydraulic state of flow. The hydraulic jump length over a smooth horizontal bed depends acceleration due to gravity g, depth of upstream flow y_1, average velocity of upstream flow u_1, depth of downstream flow y_2 and cinematic viscosity of fluid ν.

$$L_j = f(y_1, u_1, g, y_2, \nu) \quad (5)$$

By applying the principles of dimensional analysis (Buckingham π theorem), the following relationship is obtained:

$$\frac{L_j}{y_1} = \phi(\frac{u_1 y_1}{\nu}, \frac{u_1}{\sqrt{g y_1}}, \frac{y_2}{y_1}) \quad (6)$$

Where $\frac{u_1}{\sqrt{y_1 g}}$ is upstream Froude number at the beginning of the hydraulic jump and $\frac{u_1 y_1}{\nu}$ is the Reynolds number of approaching flow. For the large value of the Reynolds number, viscous effects can be neglected [32]. As a result the final equation is derived as follow:

$$\frac{L_j}{y_1} = \varphi(Fr_1, \frac{y_2}{y_1}) \quad (7)$$

2-2-Experimental data

The classical hydraulic jump data was gathered from USBR reports and published data from literature[2,3,33]. The experiments have been done on 6 different flumes. The range of data parameters has presented in Table 1.Upstream Froude number is one of the most important parameters. Table 1 shows that wide range of Froude number has been covered in experimental data. Total number of experimental data was 167.

Table 1: Range of experimental data

Parameter	Range
Width of flumes (m)	1.5,0.61,0.5,1.21,0.3
Maximum discharge (l/s)	170,340,142,793,283,142
Upstream depth of jump(cm)	1-10
Downstream depth of jump(cm)	12-117
Upstream Froude number	1.7-19.52

2-3-TLBO algorithm

Teaching-learning based optimization algorithm (TLBO) which is based on the natural phenomenon of teaching and learning was introduced firstly byRao and et al[34, 35]. One of the most important advantages of the TLBO algorithm over the other meta-heuristic algorithms, such as artificial bee colony (ABC), particle swarm optimization (PSO) and ant colony optimization, are its simplified numerical algorithm and its independence on numerous control parameters to define the algorithm's performance [36]. Thus TLBO algorithm provides more accurate and global optimum solutions and it consumes less time compared to other optimization methods such as ABC and PSO algorithms [26].

Teaching-learning based optimization algorithm has two control parameters which are the population size (student number) and maximum number of iteration. Like other optimization methods, it uses randomly generated initial population size [35].

The process of working of TLBO is divided into two parts. Teacher phase which means learning from teacher and learner phase which means learning through the interaction between learners.

The algorithm steps are as follows [35]:

1- Preparation: Initialize population size (number of students) and maximum number of iteration (termination criteria)
2- Calculate the mean of each variable
3- Identify the best solution (as teacher)
4- Teaching phase: Modify solution based on best solution

$$Difference_{Mean\ i} = r_i(M_{new} - T_F M_i) \quad (8)$$

Where $X_{teacher}$ is desired mean, M_i is current mean, r_i is a random number between [0,1] andT_F is a teaching factor (its value can be either 1 or 2) that it can be calculated as follow:

$$T_F = round[1 + rand(0,1)\{1,2\}] \quad (9)$$

The modification of existing solution is according to the following equation:

$$X_{new,i} = X_{old,i} + Difference\_Mean_i \quad (10)$$

5- Learner phase: In the learning phase, modified students increase their knowledge by means of interaction between each other according to teaching–learning process [36]. A learner collaborates arbitrarily with different learners with the aim of group discussion, communication, etc. A learner learns something new if the other learner has more knowledge than him or her. Learner modification is expressed as [35],

$For\ i = 1: P_n$

Randomly select another learner X_j, such that $i \neq j$

$If\ f(X_i) < f(X_j)$

$X_{new,i} = X_{old,i} + r_i(X_i - X_j)$

$Else$

$X_{new,i} = X_{old,i} + r_i(X_j - X_i)$

$End\ if$

$End\ for$

Accept X_{new} if it gives a better function value

The new student obtained from student phase is not taken into account if its objective function is not better, as well as in the teaching phase. At the end of the learning phase, a cycle (iteration) is completed for the TLBO then learning and teaching phases are continued until reaching a termination criterion [36].

Detailed description of the TLBO algorithm and its implementation are given by Rao, Savsani [35].

2-4-Statistical criteria

The objective function for the teaching based learning algorithm is sum of square error (SSE) given by the following equation:

$$min \rightarrow SSE = \sum_{i=1}^{N}(P_i - O_i)^2 \quad (11)$$

Where N= number of observations, O_i = observed value, and P_i= predicted value of the regression functions.

To estimate the accuracy of the proposed models the following expressions were used:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(O_i - P_i)^2}{N}} \quad (12)$$

$$MBE = \frac{1}{N}\sum_{i=1}^{N}(O_i - P_i) \quad (13)$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{O_i - P_i}{O_i}\right| \quad (14)$$

$$R^2 = \frac{(\sum_{i=1}^{N}(O_i - \bar{O}_i)(P_i - \bar{P}_i))^2}{\sum_{i=1}^{N}(O_i - \bar{O}_i)^2 \sum_{i=1}^{N}(P_i - \bar{P}_i)^2} \quad (15)$$

Where O_i is the observed value, P_i is the predicted value, $\bar{O}_i$ is the mean value of observations, $\bar{P}_i$ is the mean value of predictions, i is the subscript which indicates the ID of data, and N is the total number of data. Best model was selected according to minimum RMSE and maximum R^2.

3-Results and discussion

3-1-Results

From 167 data sets, 117 (70 percent) data sets were used for TLBO regression model training and the rest of the data for testing. Train and test data were selected randomly. In the Modeling process, four regression functions (quadratic, exponential, linear and power functions) are used to predict hydraulic jump length over a horizontal smooth bed. Then the TLBO algorithm is used to optimize coefficients of regression functions. One of the most important challenges of the met-heuristic optimization algorithms like TLBO is to set the parameters of the algorithms. The control parameters of TLBO algorithm were chosen as follows: number of maximum iteration (termination criteria) = 10,000 and size of initial population (number of students) = 50, 100, 150 and 200, respectively. TLBO algorithm parameter ranges were between [-30, 30]. Using TLBO algorithm to optimize the coefficients of different regression functions, following equations obtained:

$$\frac{L_j}{y_1} = 2.84 - 12.597Fr_1 + 14.908\frac{y2}{y_1} \quad (16)$$

$$\frac{L_j}{y_1} = 3.987. Fr_1^{-1.9951}.\left(\frac{y_2}{y_1}\right)^{2.9081} (17)$$

$$\frac{L_j}{y_1} = -13.558 + 11.621 Fr_1 - 0.326.\frac{y2}{y_1} + 12.59. Fr_1.\left(\frac{y2}{y_1}\right) - 10.07. Fr_1^2 - 3.95.\left(\frac{y2}{y_1}\right)^2 (18)$$

$$\frac{L_j}{y_1} = 10 + \exp(3.529 - 0.055. Fr_1 + 0.105.\left(\frac{y2}{y_1}\right))(19)$$

Results of regression models, empirical equations and observed values are compared and the best fitted equations are determined. The comparison is made using statistical performance indices, i.e. RMSE, MAE, MABE and R^2. The performance indices for testing set are presented in Table 2. The best fitted equation is marked bold. According to Table 2 the best regression equation is obtained from the quadratic function (RMSE= 0.164 m and R^2=0.974). The second high precision model is Linear TLBO model (RMSE= 0.169 m and R^2=0.975). As can be seen from Table 2, all TLBO regression models predict the hydraulic jump length better than proposed empirical models [1, 7]. Comparison of two empirical models of Silvester [7] and Carollo, Ferro [1] shows that Silvester [7] model (RMSE=0.301 m and R^2=0.945) give better results for our dataset. Carollo, Ferro [1] model underestimate the hydraulic jump length. Thus, the developed TLBO regression models can predict the target values of the hydraulic jump length with acceptable accuracy and less error than the available models. It can be interpreted that this is because the quadratic model together with various mathematical computing methods such as $x_1 \times x_2$ creates new independent variables by using available independent variables and in this way the precision of the model increases.

Table 2: The model results for test data

Model	RMSE(m)	R^2	MAPE (%)	MBE(m)
Quadratic TLBO	**0.164**	**0.974**	**6.4**	**0.015**
Linear TLBO	0.169	0.975	6.88	0.005
Power TLBO	0.168	0.974	7.32	-0.011
Exponential TLBO	0.508	0.793	22.71	-0.108
Carollo, Ferro [1]	1.16	0.933	52.12	1.09
Silvester [7]	0.301	0.945	11.09	-0.178

Figure 2 illustrates a comparison of the observation results with the computed results from testing sets. Although determination of the fittest equation from Figure 2 difficult, the error values mentioned above shows that the model with TLBO algorithm has higher accuracy.

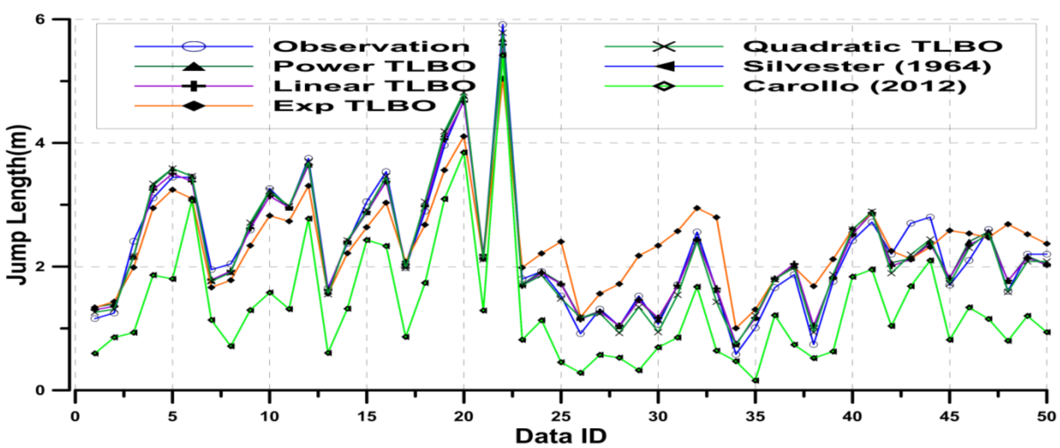

Figure 2: The comparison of the observed jump length values with the predicted ones for different models

Figures 3, 4 and 5 also provide a different illustration of the performance for the best fitting model for testing sets. The nearer the points gather around the diagonal, thebetter the learning results are. The relative errors of the points on the diagonal are zero.

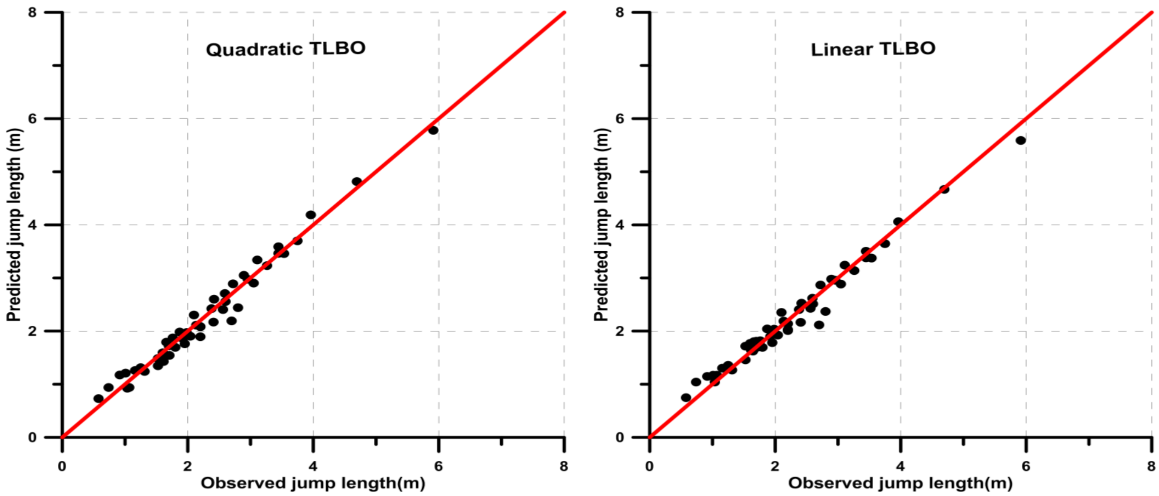

Figure 3: Comparison of observed and predicted values of quadratic and linear TLBO models

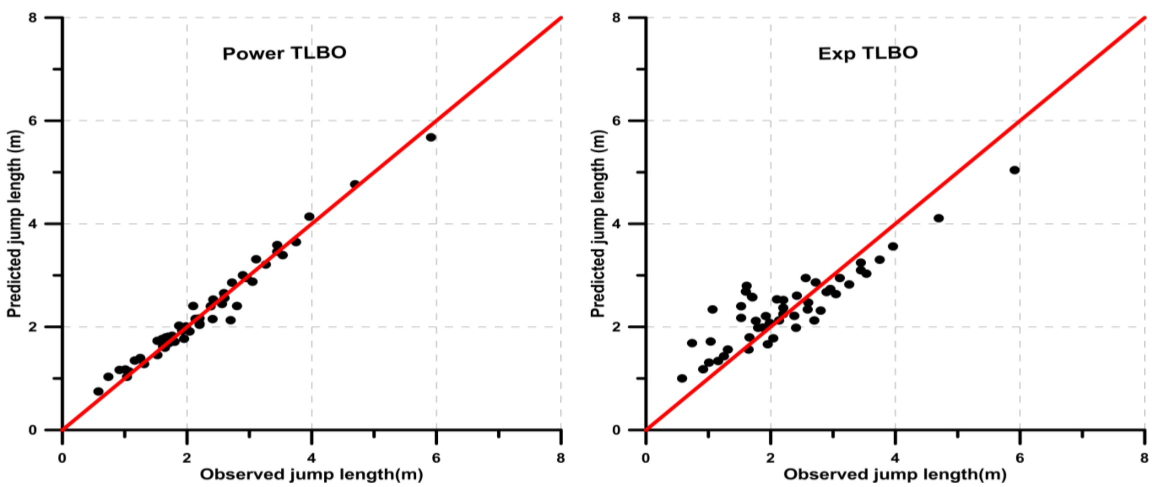

Figure 4: Comparison of observed and predicted values of power and exponential TLBO models

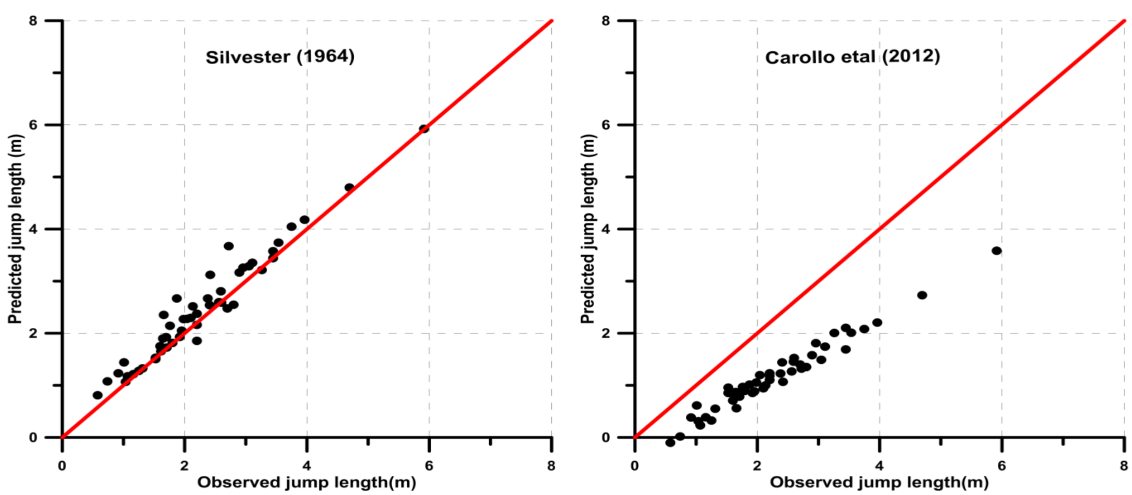

Figure 5: Comparison of observed and predicted values of Silvester [7] and Carollo, Ferro [1]models

3-2-Uncertainty analysis for developed TLBO models

In this section, a quantitative evaluation of the uncertainties in the prediction of the jump length is presented employing the available existing equations, and the developed TLBO models. The uncertainty analysis describes the model prediction error as $e_i = P_i - O_i$. The calculated model prediction errors for the dataset are utilized to calculate the mean and standard deviation of the prediction errors as $\bar{e} = \sum_{i=1}^{n} e_i$ and $S_e = \sqrt{\sum_{i=1}^{n}(e_i - \bar{e})^2/n-1}$ respectively. A negative mean value implies that the prediction model underestimated the observed values, and a positive value implies that the model overestimated the observed values [37]. Using the values of $\bar{e}$ and S_e, a confidence band can be defined over the predicted values of an error using the Wilson score method without continuity correction, the use of $\pm 1.96 S_e$yields an approximately 95% confidence band [37]. Table 3, shows the mean prediction errors of the different models, the width of the uncertainty band, and the 95% prediction interval error.

Table 3: Uncertainty Estimates for different models

Model	Mean prediction error $\bar{e}$	S_e	Width of uncertainty band$\pm 1.96 S_e$
Quadratic TLBO	-0.016	0.165	$\pm$0.323
Linear TLBO	-0.005	0.172	$\pm$0.336
Power TLBO	0.011	0.170	$\pm$0.333
Exponential TLBO	0.109	0.501	$\pm$0.983
Carollo, Ferro [1]	-1.098	0.381	$\pm$0.746
Silvester [7]	0.178	0.244	$\pm$0.478

According to Table 6, all TLBO models have mean prediction error for jump length less than available equations ([1, 7]). Mean prediction error of quadratic and power models is less otherTLBO models. The uncertainty band for the quadratic model ranged $\pm$0.323 for hydraulic jump length. This range was smaller than those of available equations ([1, 7]). The uncertainty band for the power model was slightly smaller than linear model.

Conclusions

In this study, the ability of TLBO algorithm to model hydraulic jump length based on upstream Froude number (Fr_1) and ratio of (y_2/y_1) is investigated. The conclusions of present study can be summarized as follows:

- The results of the TLBO based models had good agreements with the measured experimental data.
- The comparison between the results shows that the best fit equation for hydraulic jump length prediction is obtained from the quadratic function
- Quadratic, Linear and power TLBO models have higher accuracy in comparison of empirical equations that had been proposed by other researchers.
- The application of the TLBO algorithm by hydraulic engineers is recommended for future studies since the results of the proposed model are found to be satisfactory for this study.

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