



Calculation on Stability and Common Proportions of the New Retaining Wall Formed from Cylindrical Shells

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Abstract: Retaining walls are structures used to provide stability for earth or other material where conditions disallow the mass to assume its natural slope and are commonly used to hold back or support soil banks coal or ore piles, and water. Retaining walls must be of adequate proportions to resist overturning (or excessive tilting) and sliding as well as being structurally adequate. Recognition of retaining walls behavior, selection of the best and the most economical design, and the most logical form and their required specifications are the most important issues and worries of designing engineers during designing period. So recognition of the main effective elements on wall construction expenses, and the effect of limitations on acquiring the most economical wall can be a proper help and deserves being completely studied from all points of view in order to acquire the most economical and from the point of technical aspect the most suitable one. According to the elements of design resulting from the theoretical researches on former retaining walls, and for lightening them; in this article a new constructing design of lightened retaining walls is suggested. Meanwhile common proportions and stability of multi cylindrical shell retaining wall have been discussed in this paper. In this article, ANSYS computer program has been used for the numerical simulation of the new retaining wall using the finite element method. Also, the amount of concrete used for the new retaining wall has been compared with the existing retaining walls.

1. Introduction

Retaining wall design proceeds with the selection of tentative dimensions, which are then analyzed for stability and structural requirements and are revised as required. Since this is a trial process, several Solutions to the problem may be obtained, all of which are satisfactory. A computer solution greatly simplifies the work in retaining wall design and provides the only practical means to optimize the design (Li *et al.*, 2020; Shakeel *et al.*, 2022). Mass concrete or masonry walls rely largely on their weight for stability against overturning and sliding (Sari *et al.*, 2020). They are unreinforced so their height must be limited to ensure internal stability of the wall in bending and shear when subjected to the lateral stresses. They are typically no more than about 3m high. Providing a minimum slope of 1:50 (horizontal: vertical) on the front face avoids the illusion of a vertical wall tilting for wards. Reinforced concrete walls are more economical with the reinforcement enabling the stem and base sections to be designed as cantilevered structural elements. Overall stability is provided by an adequate base width and the weight of backfill resting on the base slab behind the stem.

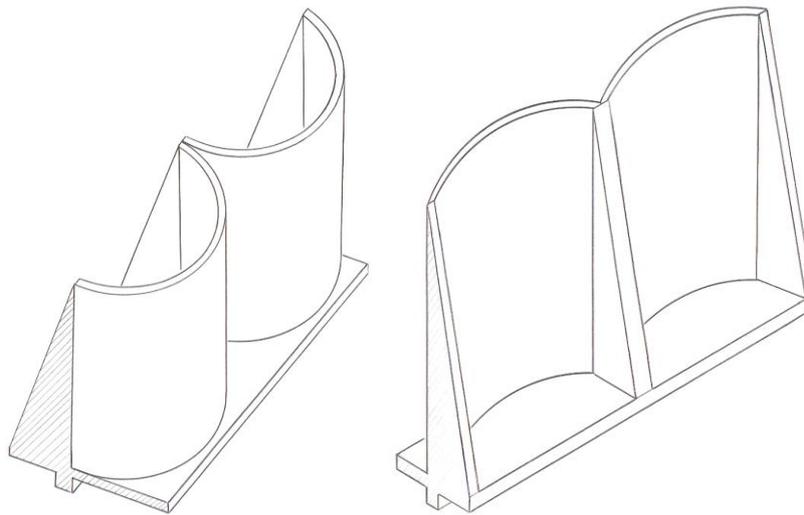


Figure 1. Multi cylindrical shell retaining wall

As **Figures 1** illustrates the retaining walls resulted will be considerably economical regarding consumption of material. In other words, for front bearing element of retaining wall, multi cylindrical shells and for base element (foundation), flat slab is taken into consideration. As mentioned, for front bearing element against soil pressure, multi cylindrical concrete shell is used instead of flat concrete slab because shells are more suitable than flat slabs since they are thinner than flat slabs when facing equal pressures, so their use is more economical regarding material consumption.

Surveys have shown that the use of curved shells compared to flat slabs saves 25-30% in material consumption ([Gupta *et al.*, 2003](#); [Raju *et al.*, 1988](#)).

The required thickness for the new shell retaining wall is less than the required thickness for the counterfort retaining wall, and this is due to the quality of the behavior of the shells against the incoming forces compared to flat slabs. From a structural point of view, the shells are superior to the slabs, because in the shells, the entire cross-section is uniformly subjected to axial stresses and the bending stresses can be ignored, that is why the thickness of the shells is usually small. Therefore, the use of shell retaining walls will be cost-effective.

Many books on shells only give the key equations or snippets of theory, skipping all of the mathematical steps required to solve for the key equations. This is understandable, because of the mathematical complexity of shell structures. Thus, the reader must just accept the design equations blindly, without achieving a complete understanding of shell theory. There are many texts related to the analysis of shells, a number of selected and studied sources in this regard are given in the references section ([Ciarlet, 2022](#); [Gohnert, 2022](#)).

Geotechnical engineering is now a fundamental component of construction projects. A number of selected and studied texts regarding geotechnical engineering and retaining walls are given in the references section ([Chang-Yu Ou *et al.*, 2023](#); [Dhouib, 2023](#)). This paper presents a new retaining structure for soil reinforcement and improvement. This article presents both theoretical explication and practical applications for readers to easily comprehend the theoretical background, design methods, and practical applications and considerations.

Many studies have been done by the author of this article on retaining walls ([Majidpourkhoei, 2016](#); [Majidpourkhoei, 2020](#)). However, the results of studies regarding the styling of retaining walls in the proposed form are presented for the first time in this article. In this article, according to the above explanations, the shells are matched with retaining walls and the use of new shell retaining walls is suggested.

2. Methodology

2.1 Common proportions of multi cylindrical shell retaining wall

Dimensions for a retaining wall should be adequate for structural stability and to satisfy local building code requirements. The tentative dimensions shown in **Figures 2** and **Figures 3** are based in part on the history of satisfactorily constructed walls, and may be used in the absence of other data, but may result in an overly conservative design. These dimensions are only a guide, and thinner wall sections may be used if structural stability is satisfied. The use of a counterfort multi cylindrical shell retaining will be determined by the relative costs of forms, concrete, reinforcing and labor. It is doubtful if a counterfort wall will provide any relative construction economy unless it is over 10 m in height. The spacing of the counterforts is a process to give a minimum cost. The most economical spacing appears to be from one-third to one-half the height of the wall. A counterfort may be built into the beginning of the wall or by allowing a part of the wall to overhang. The overhanging configuration may prove to be more economical, since this saves the concrete and from work on the two counterforts at the joint. The counterfort multi cylindrical shell retaining wall may be constructed without a toe if additional front clearance is needed and sliding and overturning stability requirements are met. Multi cylindrical shell retaining walls are most economically designed considering the wall and heel as plates fixed on their edges and using a finite element or finite difference computer program. One of the main procedures related to designing of shell structure is the selection of geometrical shape and force transfer system design by shell structure. Considering the generalities of structure and the relation of main shell with other elements is one of the important considerations in shell structure designing. This is important because of behavior, balance and general stability of structure specifically the performance of structure against forces. These points are also considered in proposed structure for retaining wall. According to soil lateral force theories, wall shell will be influenced by a triangular load resulted from soil active pressure (**Figures 4**).

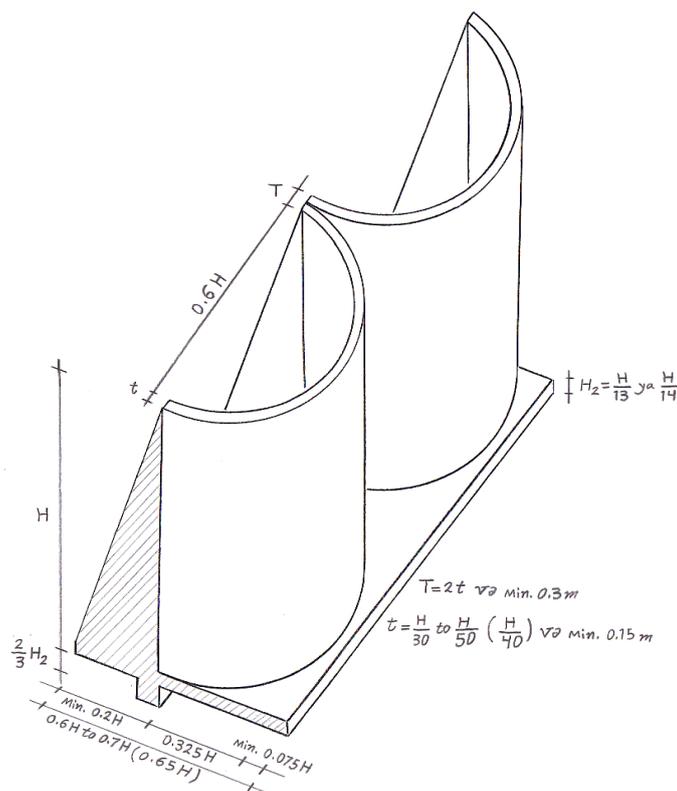


Figure 2. Tentative design dimensions for a multi cylindrical shell retaining wall

Also, general proportions (Measures) of proposed retaining wall are regarded the same as [Figures 2](#) and [Figures 3](#). The connection of wall shell with base slab is assumed rigid. Also, the base slab is considered far thicker than cylindrical shells. Further the rigidity of base slab flexure is considered much. The linkage place of cylindrical shell with base slab against torsion has much rigidity. Considering all these factors, the linkage place of base slab with cylindrical shells is rigid and restrained.

2.2 Stability of multi cylindrical shell retaining wall

The general stability of lightened shell retaining walls is studied. The retaining walls should satisfy the condition for stability against overturning, stability against sliding and stability for foundation base as discussed below.

2.2.1 Stability against overturning

The retaining wall shall not overturn when restoring moment is greater than the overturning moment. The restoring moment due to live load and vertical component of active earth pressure may be neglected. The overturning moment is caused by the horizontal component of active earth pressure it should possess adequate factor of safety given by ([Barns, 2000; Bowels et al., 1977; Das et al., 1990](#)):

$$F.S_{(\text{overturning})} = \frac{\sum M_R}{\sum M_O} \quad \text{Eqn. 1}$$

$$M_O = P_h \left(\frac{H'}{3} \right) \quad \text{Eqn. 2}$$

2.2.2 Stability against sliding

The retaining wall has the tendency to slide under the action of horizontal component of active earth pressure which is resisted by the frictional force acting at the base of retaining wall and the passive earth pressure. Generally the passive earth pressure is small if the height of earth in front of retaining wall is small and neglected.

The retaining wall should possess adequate factor of safety against sliding which is given by ([Barns, 2000; Bowels et al., 1977; Das et al., 1990](#)):

$$FS_{(\text{sliding})} = \frac{(\sum V) \tan \alpha_2 + B.d.c_2 + P_p}{P_a \cos \delta} \quad \text{Eqn. 3}$$

$$P_p = \frac{1}{2} K_p \gamma_2 D^2 .d + 2C_2 \sqrt{K_p} D.d \quad , \quad D=H_3 \quad \text{Eqn. 4}$$

$$K_p = \tan^2(45 + \alpha_2/2) \quad \text{Eqn. 5}$$

2.2.3 Stability of foundation base

The foundation base should be under compression for which the resultant R of total vertical load W and active earth pressure P_a must lie within middle third of the base. The maximum upward soil pressure should be within the permissible bearing capacity of soil ([Bowels et al., 1977; Das et al., 1990; Tahoni, 1995](#)):

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V}, \quad e < \frac{B}{6} \quad \text{Ok.} \quad \text{Eqn. 6}$$

$$q_{\max} = q_{toe} = \frac{\sum V}{B.d} \left(1 + \frac{6e}{B}\right) \quad \text{Eqn. 7}$$

$$q_{\min} = q_{heel} = \frac{\sum V}{B} \left(1 - \frac{6e}{B}\right) \quad \text{Eqn. 8}$$

$$q_u = c_2 N_c F_{Cd} F_{Ci} F_{Cs} + q N_q F_{qd} F_{qi} F_{qs} + \frac{1}{2} \gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i} F_{\gamma s} \quad \text{Eqn. 9}$$

$$q = \gamma_2 \cdot D \quad \text{Eqn. 10}$$

$$N_q = \tan^2 \left(45 + \frac{\alpha_2}{2}\right) e^{\pi \cdot \tan \alpha_2} \quad \text{Eqn. 11}$$

$$N_c = (N_q - 1) \cot \alpha_2 \quad \text{Eqn. 12}$$

$$N_\gamma = 2(N_q + 1) \tan \alpha_2 \quad \text{Eqn. 13}$$

$$B' = B - 2e \quad \text{Eqn. 14}$$

$$F_{cd} = 1 + 0.4 \left(\frac{D}{B'}\right) \quad \text{Eqn. 15}$$

$$F_{qd} = 1 + 2 \tan \alpha_2 (1 - \sin \alpha_2)^2 \left(\frac{D}{B'}\right) \quad \text{Eqn. 16}$$

$$F_{\gamma d} = 1 \quad \text{Eqn. 17}$$

$$\psi^\circ = \tan^{-1} \left(\frac{P_a \cos \alpha}{\sum V}\right) \quad \text{Eqn. 18}$$

$$F_{Ci} = F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2 \quad \text{Eqn. 19}$$

$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi^\circ}\right)^2 \quad \text{Eqn. 20}$$

$$FS_{(\text{bearing capacity})} = \frac{q_u}{q_{\max}} > 3 \quad \text{Ok.} \quad \text{Eqn. 21}$$

2.3 Citing international regulations and standards

The design of shell structures is usually done with the help of general and codified rules called shell standards and regulations. Among them, the American ACI regulations regarding silos, the American regulations regarding concrete tanks, the 19th chapter of the ACI regulations regarding concrete shells can be mentioned ([ACI 318-77](#); [ACI 334](#); [ACI 318-83](#); [ACI 350R-83](#); [ACI 313-77](#)).

In terms of strength, the thickness of the shell should be chosen so that there is no possibility of buckling in pressure areas. In terms of performance, the thickness of the shell should be so much that it can accommodate different layers of reinforcement and at the same time provide sufficient coverage for the reinforcements on both sides. Different standards and regulations each prescribe a minimum thickness for shells. In general, it can be concluded that the minimum thickness for shell roofs is usually between 80 and 120 mm ([Timoshenko et al., 1959](#); [Ugural, 1981](#)).

According to the regulations of India, the minimum thickness of the wall shell in concrete liquid tanks is equal to the following value (Raju *et al.*, 1988):

$$T = 25\text{mm} + \frac{H}{40} \quad \text{Eqn. 22}$$

H is the height of the free surface of the water to the desired section in millimeters.

Anyway, the author, after reviewing the regulations of similar structures such as tanks and silos and adapting it to the proposed retaining wall, and taking into account the implementation factors and stability control, suggests the following values for the thickness of the retaining wall shell (H is the height of the retaining wall):

$$t = \left(\frac{H}{30} \text{ to } \frac{H}{50} \right) = \frac{H}{40} \quad , \quad t_{\min} = 150 \text{ mm} \quad \text{Eqn. 23}$$

3. Results and Discussion

3.1 Numerical example of the stability of multi cylindrical shell retaining wall

Numerical examples of solving problem by choosing numerical values for physical and geometrical parameters of cylindrical shells are made. Computations are represented for the following figure geometrical quantities (Figures 5).

The coefficient of active earth pressure for sloping backfill is given by:

$$C_a = 0.337$$

$$H' = H_1 + H_2 + H_3$$

$$H' = 1.2 \tan 5^\circ + 11.15 + 0.85 = 12.105 \text{ m}$$

The total active earth pressure acting on unit length of wall is determined as follows:

$$P_a = \frac{1}{2} \gamma_1 \cdot H'^2 \cdot C_a \times 7.8 \quad \text{Eqn. 24}$$

$$P_a = (7.8/2) \times 18 \times (12.105)^2 \times 0.337 = 3466.543 \text{ KN}$$

$$P_v = P_a \sin 5^\circ = 3466.543 \sin 5^\circ = 302.129 \text{ KN}$$

$$P_h = P_a \cos 5^\circ = 3466.543 \cos 5^\circ = 3453.352 \text{ KN}$$

Stability against overturning:

$$M_o = P_h \left(\frac{H'}{3} \right) = 3453.352 (12.105/3) = 13934.275 \text{ KN.m}$$

$$\gamma_{\text{beton}} = 23.58 \text{ KN/m}^3 \quad , \quad \gamma_{\text{torpaq}} = 18 \text{ KN/m}^3$$

$$F.S.(\text{overturning}) = \frac{\sum M_R}{\sum M_O} = \frac{33197.639}{13934.275} = 2.38 > 1.5 \quad \text{Ok.}$$

Stability against sliding:

$$FS(\text{sliding}) = \frac{(\sum V) \tan \alpha_2 + B.d.c_2 + P_p}{P_a \cos \delta}$$

$$P_p = \frac{1}{2} K_p \gamma_2 D^2 .d + 2C_2 \sqrt{K_p} D.d \quad , \quad D = H_3 = 0.85 \text{ m}$$

$$K_p = \tan^2(45 + \alpha_2/2) = \tan^2(45 + 15) = 3$$

$$P_p = (1/2)(3)(19)(0.85)^2(7.8) + (2)(40)(\sqrt{3})(0.85)(7.8) = 918.679$$

$$FS_{(\text{sliding})} = \frac{(5863.296) \tan(30) + (7.8)(7.8)(40) + (918.679)}{3453.352} = 1.95 > 1.5 \quad \text{Ok.}$$

Stability of foundation base:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} = \frac{7.8}{2} - \frac{33197.639 - 13934.275}{5863.296} = 0.6146 \text{ m}$$

$$e = 0.6146 < \frac{B}{6} = \frac{7.8}{6} = 1.3 \quad \text{Ok.}$$

$$q_{\max} = q_{\text{toe}} = \frac{\sum V}{B \cdot d} \left(1 + \frac{6e}{B}\right) = \frac{5863.296}{7.8 \times 7.8} \left(1 + \frac{6 \times 0.6146}{7.8}\right) = 141.934 \text{ KN/m}^2$$

$$q_{\min} = q_{\text{heel}} = \frac{\sum V}{B} \left(1 - \frac{6e}{B}\right) = \frac{5863.296}{7.8 \times 7.8} \left(1 - \frac{6 \times 0.6146}{7.8}\right) = 50.81 \text{ KN/m}^2$$

$$q_u = c_2 N_c F_{cd} F_{ci} F_{cs} + q N_q F_{qd} F_{qi} F_{qs} + \frac{1}{2} \gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i} F_{\gamma s}$$

$$q = \gamma_2 \cdot D = 19 \times 0.85 = 16.15 \text{ KN/m}^2$$

$$N_q = \tan^2 \left(45 + \frac{\alpha_2}{2}\right) e^{\pi \cdot \tan \alpha_2} = 18.40$$

$$N_c = (N_q - 1) \cot \alpha_2 = 30.14$$

$$N_\gamma = 2(N_q + 1) \tan \alpha_2 = 22.40$$

$$B' = B - 2e = 7.8 - 2(0.6146) = 6.571 \text{ m}$$

$$F_{cd} = 1 + 0.4 \left(\frac{D}{B'}\right) = 1 + 0.4 \left(\frac{0.85}{6.571}\right) = 1.0517$$

$$F_{qd} = 1 + 2 \tan \alpha_2 (1 - \sin \alpha_2)^2 \left(\frac{D}{B'}\right) = 1 + 2 \tan 30 (1 - \sin 30)^2 \left(\frac{0.85}{6.571}\right) = 1.0373$$

$$F_{\gamma d} = 1$$

$$\psi^\circ = \tan^{-1} \left(\frac{P_a \cos \alpha}{\sum V}\right) = \tan^{-1} \left(\frac{3453.352}{5863.296}\right) = 30.4968$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2 = \left(1 - \frac{30.4968}{90^\circ}\right)^2 = 0.437$$

$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi^\circ}\right)^2 = \left(1 - \frac{30.4968}{30^\circ}\right)^2 = 0.00027$$

$$q_u = (40) \times (30.14) \times (1.0517) \times (0.437) \times (1) + (16.15) \times (18.40) \times (1.0373) \times (0.437) + (0.5) \times (19) \times (6.571) \times (22.40) \times (0.00027) = 689.165$$

$$FS_{(\text{bearing capacity})} = \frac{q_u}{q_{\max}} = \frac{689.165}{141.934} = 4.85 > 3 \quad \text{Ok.}$$

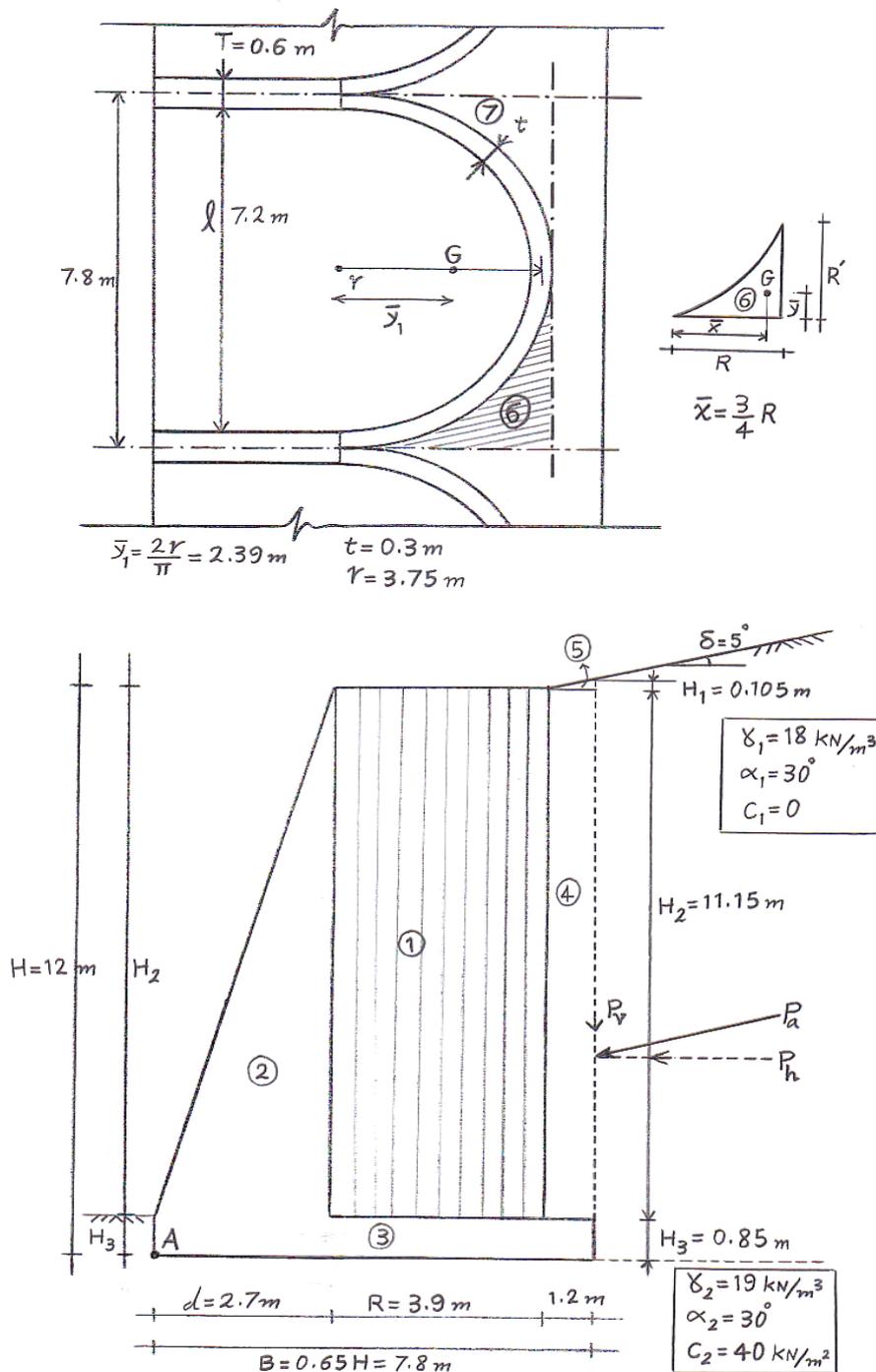


Figure 5. Geometrical parameters of cylindrical shell retaining wall

3.2 Numerical simulation of new retaining wall by finite element method

Considering the complexity of the structure, ANSYS computer program has been used to numerically model this problem. In this research, the following are considered for solving and analyzing the new retaining wall by the finite element method:

- 1- In the modeling of the new retaining wall using the ANSYS program, a three-dimensional shell element with four nodes has been determined and used.
- 2- The modulus of elasticity of concrete $E_b = 2 \times 10^{10} \text{ N/m}^2$ is considered.
- 3- In solving the problem, Poisson's ratio of concrete is considered to be 0.2

4- In each of the structural elements, there are four nodes and six degrees of freedom in each node, which includes three translational degrees of freedom in the direction of the Z, Y, X axes and three rotational degrees of freedom around the Z, Y, X axes.

5- According to the position and location of each element in the structure, the degrees of freedom of the nodes are considered to be some free and some closed by means of the ANSYS computer program.

6- By considering and accepting three translational degrees of freedom for a four-node shell element in the direction of the X, Y, and Z axes, the obtained stiffness matrix will be a 12x12 matrix, which is determined by the ANSYS computer program for each element.

7- The external degrees of freedom of the shell retaining wall, including the translational movement of the upper edge of the shell in the direction of the Z axis and the rotational movement around the X axis, are considered free, and the rest of the external degrees of freedom of the structure are considered closed (Figures 6).

In Figures 7 and Figures 8, the results of solving the new retaining wall using the finite element method are presented.

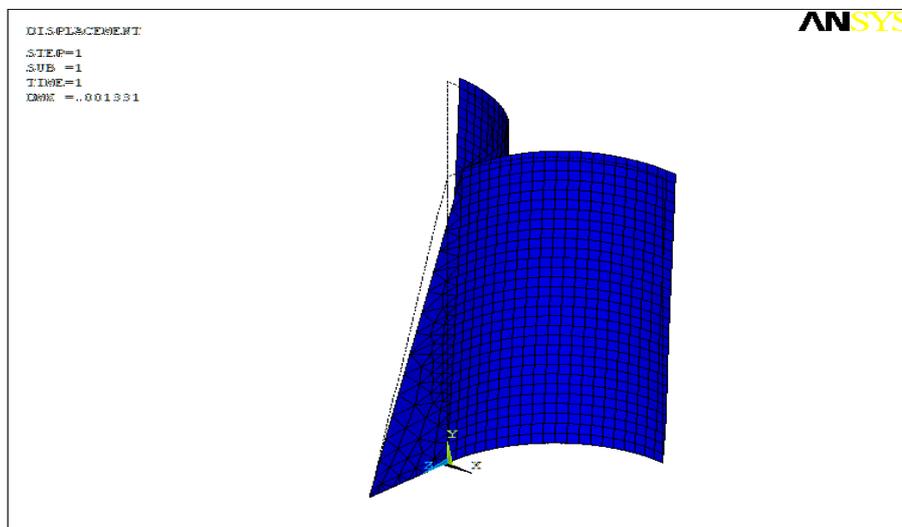


Figure 6. Structure elements for the finite element method

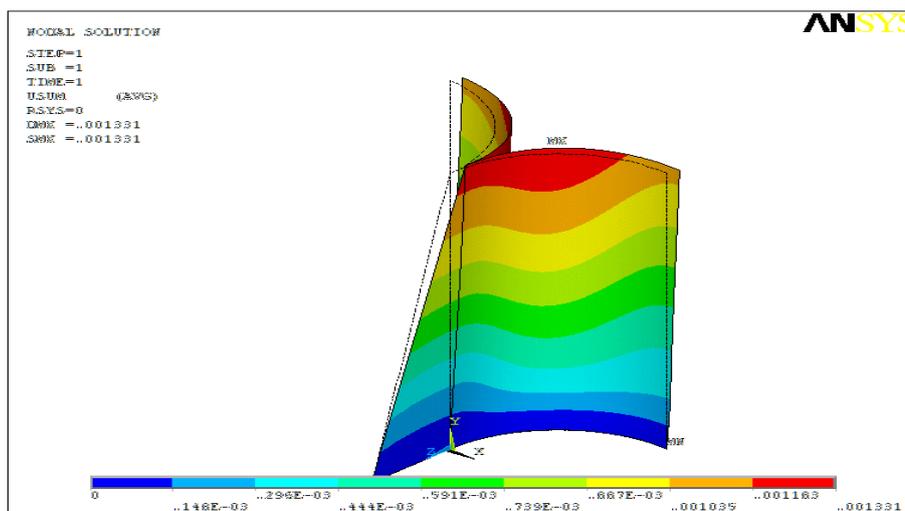


Figure 7. Displacement of cylindrical shell retaining wall

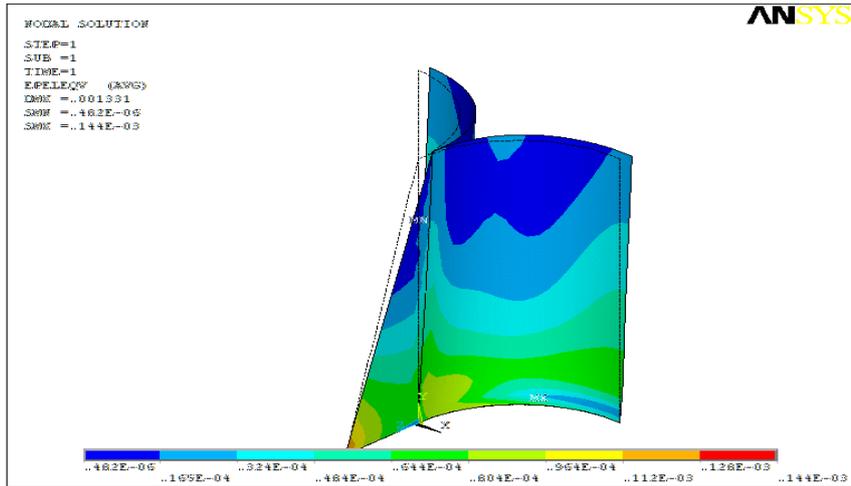


Figure 8. Deflection of cylindrical shell retaining wall

3.3 Comparison of the volume of concrete used for the new retaining wall with the existing retaining walls

The dimensions considered for comparing the volume of concrete used for the retaining wall are according to Figures 3 and Figures 9.

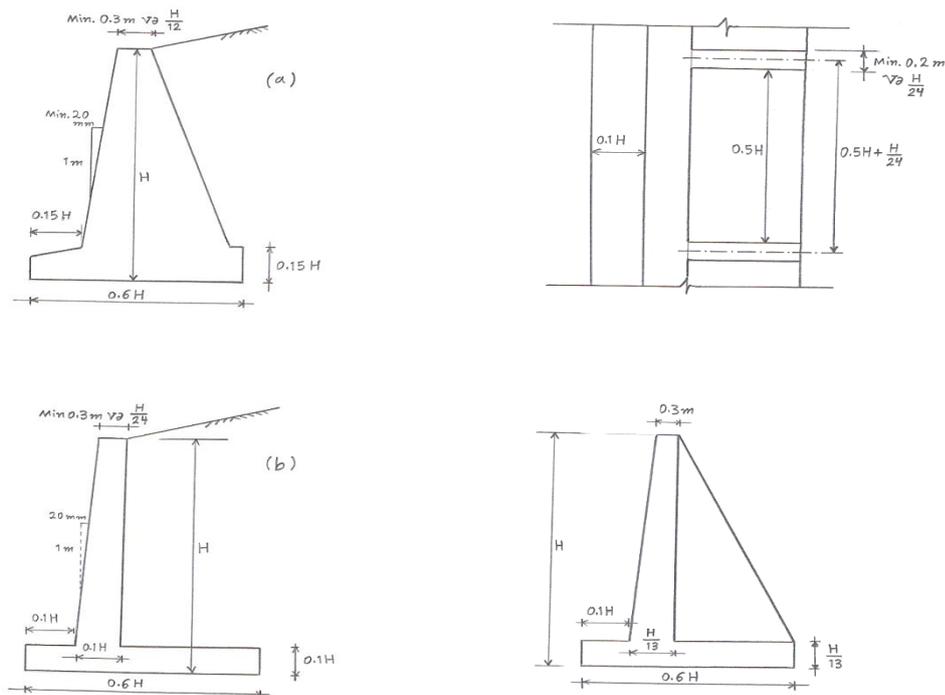


Figure 9. Existing retaining walls to compare the amount of concrete used

In order to compare the amount of concrete used for the mentioned retaining walls in this section, the considered height is between 4 and 20 meters. Based on this, the values obtained for the volume of concrete used for different types of retaining walls for different heights are compared in Figures 10. From the examination of Figures 10, it can be concluded that the use of shell retaining walls compared to counterfort and buttress retaining walls saves 14-16% in concrete consumption. Also, it saves 27-30% of concrete consumption compared to cantilever retaining walls, and 70-75% of concrete consumption compared to gravity retaining walls.

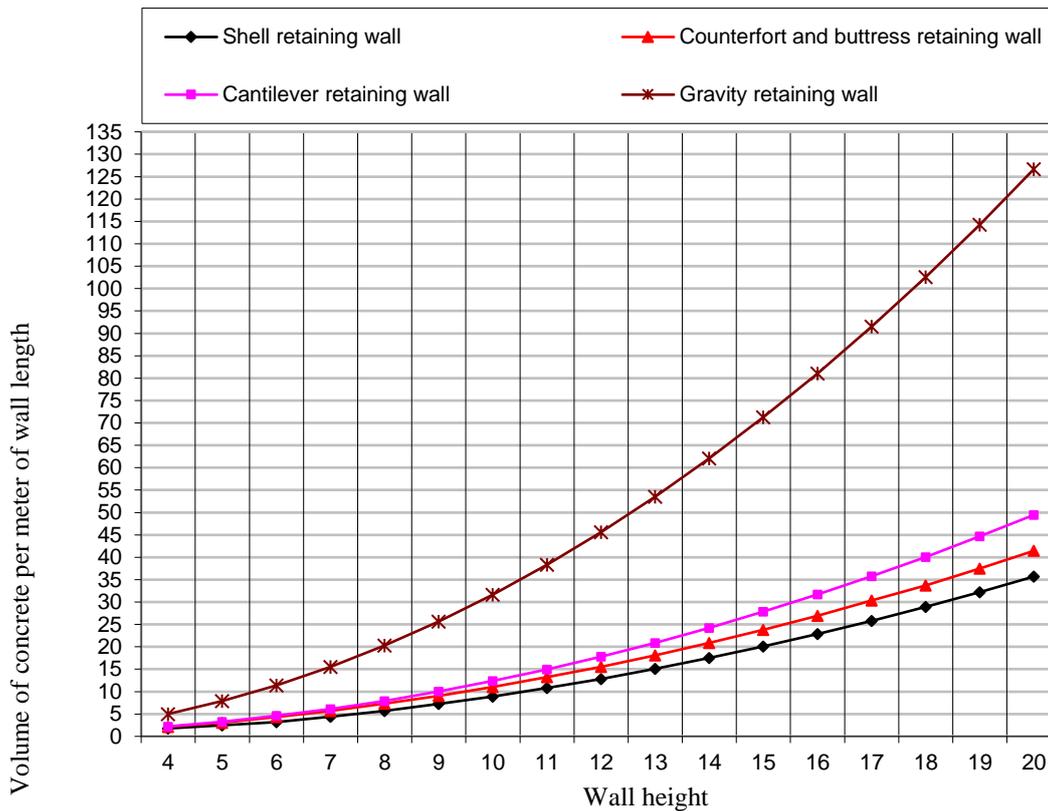


Figure 10. Comparison of the volume of concrete used for retaining walls

Conclusion

The research the results of which are presented in this paper, for the first time multi cylindrical shells are adapted with retaining walls in the proposed shape and form and numerical example of the stability of multi cylindrical shell retaining wall is stated as well.

One of the main procedures related to designing of shell structure is the selection of geometrical shape and force transfer system design by shell structure. Considering the generalities of structure and the relation of main shell with other elements is one of the important considerations in shell structure designing. This is important because of behavior, balance and general stability of structure specifically the performance of structure against forces. These points are also considered in proposed structure for retaining wall.

Multi cylindrical shell retaining structures must be designed to prevent collapse or serious damage by instability of the earth mass. The ultimate limit state of a retaining wall must be checked for: slip failure, overturning or rotational failure and bearing pressure under the toe, bearing capacity failure, translational failure or sliding. It is carried out by ensuring equilibrium conditions using the design actions and design strength of the soils. Retaining walls must provide adequate stability against sliding additional sliding stability may be derived from the use of a key beneath the base. A key into firm soil or rock may be quite advantageous, since the resistance is now the force necessary to shear the key from the base slab.

Examining the values obtained for the volume of concrete used for existing retaining walls with a shell retaining wall, shows that the use of a shell retaining wall causes significant savings in concrete consumption. The investigations carried out in this article regarding the new retaining wall indicate that the use of shell retaining walls compared to the existing counterfort and buttress retaining walls saves 14-16% in concrete consumption. Also, it saves 27-30% of concrete consumption compared to

cantilever retaining walls, and it saves 70-75% of concrete consumption compared to gravity retaining walls. Therefore, the use of the shell retaining wall proposed in this research will be more economical.

Disclosure statement: *Conflict of Interest:* The authors declare that there are no conflicts of interest.

Compliance with Ethical Standards: This article does not contain any studies involving human or animal subjects.

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