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Problem Solving of the Lightened Concrete Gravity Retaining Wall Made of Plates

Ali Majidpourkhoei [1*], Nader Shariatmadari [2]

[1]*Postdoctoral researcher, Iran University of Science and Technology, Tehran 1684613114, I.R. of Iran.*
[2]*Professor, Iran University of Science and Technology, Tehran 1684613114, I.R. of Iran.*
[*]*Corresponding author E-mail: majidpour@mail.iust.ac.ir*

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majidpour@mail.iust.ac.ir

Abstract

Types of retaining walls are used to prevent soil destruction in trenches. During the process of design, recognizing the treatment of walls, choosing the best and most economical design, the most logical form and the necessary features of walls are the major concern and the main problem of engineers. Therefore understanding the main influential features for the cost of construction and wall production and the influence of limitations for a wall, to be economical over another could be a suitable help. It is also worth to do a detailed and an extensive consideration to gain the most economical and suitable design. In this research, a new structure form to reduce the material used in concrete gravity retaining walls and a new method for analyzing the new retaining walls have been offered. In this paper based on the theory of plates and considering the physical and mathematical aspects, the equations governing the new structures provided. In solving the problem of the lightened retaining wall consisting of three flat slabs from combined principles of engineering mechanics and mathematical methods have been used and the problem has been solved.

1. Introduction

The widespread use of plate structures in construction projects makes them even more important because of their lightness, strength and cost-effectiveness. In this study, to reduce the ascending trend, the thickness of the concrete retaining walls and consequently the decrease of the ascending path of the materials consumed in relation to the height of the wall, flat slabs with retaining walls were adapted and a new type of lightweight retaining wall was proposed, then the attempt In order to establish the basis of the theory of plate and the physical and mathematical aspects of the discussion and its application in the studies carried out for the new retaining walls, the governing equations should be expanded to the extent necessary for the proposed problem state. In general, in this research, a new method based on mathematical principles is presented for calculating and analyzing a lightened retaining wall made of flat slabs.

The texts related to the analysis of the plates are numerous; some of the texts studied in this section are presented in the reference section [1-10].

Retaining wall is a structure that are designed and constructed to withstand lateral pressure of soil or hold back soil materials. The lateral pressure could be also due to earth filling, liquid pressure, sand, and other granular materials behind the retaining wall structure. There are various types of retaining wall structures which are used for numerous goals. Retaining walls is generally classified as gravity, conventional, no gravity cantilevered, and anchored. Gravity retaining walls are the walls which use their own weight to resist the lateral earth pressures. The main forces acting on gravity retaining walls are the vertical forces from the weight of the wall, the lateral earth pressure acting on the back face and the seismic loads. These forces are used herein to illustrate the design principles. If other forces are encountered, such as vehicular loads, they must also be included in the analysis. Gravity retaining wall depends on its self-weight only to resist lateral earth pressure. Commonly, gravity retaining wall is massive because it requires significant gravity load to counter act soil pressure. Sliding, overturning, and bearing forces shall be taken into consideration while this type of retaining wall structure is designed. Every time a product is created or designed to satisfy human needs, the creator tries to achieve the best solution for the task in hand and therefore performs optimization. This process is often manual, time consuming and involves a step by step approach to identify the right combination of the product and associated process parameters for the best solution. Often the manual approach does not allow a thorough exploration of the solution space to find the optimum design, resulting in suboptimal designs. Therefore, experienced engineers may be able to come up with solutions that fulfill some of the requirements on structural response, cost, aesthetics, and manufacturing but they will seldom be able to come up with the optimal structure .This study focuses on the optimum design retaining walls, as one of the familiar types of the retaining walls that may be constructed of unreinforced concrete, or reinforced concrete. The material cost is one of the major factors in the construction of gravity retaining walls therefore, minimizing the weight or volume of these systems can reduce the cost. To obtain an optimal design of such structures, this paper proposes a method.

Several studies have already been carried out by the researcher on the retaining walls consisting of three cylindrical shells [11], the new buttress retaining walls [12], a research and evaluation on the theories of cylindrical shells and the analysis methods [13], multi cylindrical shells adapted with retaining walls [14], common proportions and stability of multi cylindrical shell retaining wall [15], evaluation of the volume of concrete used in different types of retaining walls [16], multi cylindrical shell retaining wall analysis of finite element method [17], matching the bending theory of cylindrical shells with retaining walls [18], adaptation of bending theory in cylindrical shell thanks with retaining walls [19], review of development of the theory of shell analysis [20], a review of methods of shell analysis [21], a survey on types of retaining walls and forces imposed on them [22], a survey on common proportions of retaining walls and stability of retaining walls [23]. However, the results of the studies on the lightness of concrete retaining walls in the form and method proposed for the first time in this paper are presented.

2. Material and Methods

2.1. New retaining wall made of flat slabs

In this section, a new method of structural form is proposed for the concrete retaining walls (Figure 1). Also, based on the amount of concrete used, the condition is considered with the cost of its selection.

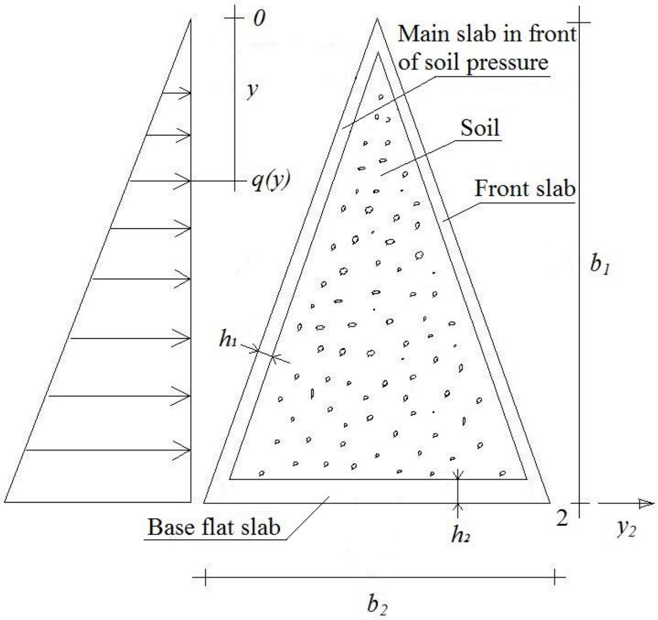

Figure 1: New retaining wall made of flat slabs.

In this paper, the general geometric proportions of the proposed retaining wall is similar to that of the concrete gravity retaining wall (Figure 2). Also, $0.7b_1 = b_2$ is proposed (b_2 is the width of the wall section and b_1 is the height of the wall).

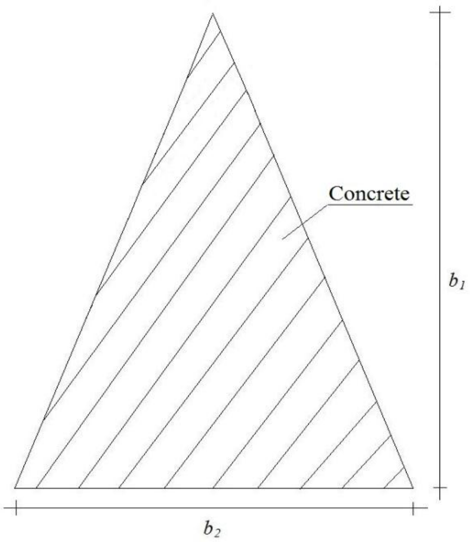

Figure 2: Concrete gravity retaining wall.

As shown in Figure 1, if the concrete gravity retaining walls space is considered to be empty and the walls of the concrete gravity retaining wall are flat slabs, and the space is filled with soil to increase stability, the resulting wall saves a significant amount of consumption concrete. Studies show that the use of a retaining wall composed of three flat slabs, compared to existing concrete gravity retaining walls, can save about 90% of concrete consumption. To achieve this, if the surface of concrete used in the concrete gravity wall is approximately $0.5b_1b_2$, this amount will be $0.04b_1b_2$ in the new retaining wall. Therefore, the use of the new retaining wall will be more cost effective.

2.2. Problem solving of the new wall

Differential equations of rectangular slabs are determined as follows. The differential equation of the main slab is written as follows:

$$D_1 \Delta^2 w_1 = q(y) \tag{1}$$

The base slab differential equation is written as follows:

$$D_2 \Delta^2 w_2 + K \cdot w_2 - K_s \left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} \right) = q^*(y) \tag{2}$$

Where D_1 and D_2, respectively, are the flexural rigidity of the main and base slabs.

$$D_1 = \frac{Eh_1^3}{12(1-\upsilon^2)} \tag{3}$$

$$D_2 = \frac{Eh_2^3}{12(1-\upsilon^2)} \tag{4}$$

Connection conditions at the location of the main and base slab joints $y_1=b_1$ and $y_2=0$ are accepted as follows:

$$w_1(b) = w_2(b) = 0 \tag{5}$$

$$\left.\frac{\partial w_1}{\partial y}\right|_{y=b_1} - \left.\frac{\partial w_2}{\partial y}\right|_{y_2=0} = 0 \tag{6}$$

$$C\frac{\partial^3 w_1}{\partial x^2\,\partial y}\bigg|_{y=b} = D_1\left(\frac{\partial^2 w_1}{\partial y^2}+\upsilon\frac{\partial^2 w_1}{\partial x^2}\right)\bigg|_{y_1=b} - D_2\left(\frac{\partial^2 w_2}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right)\bigg|_{y_2=0} \tag{7}$$

where C is the torsion stiffness of the resistant element in the location of the slab connection lines. The flexural stiffness of the resistant elements at the point of attachment of the slabs is very high ($B = \infty$), which also results in zero flexure at the location of the bonding lines of the slabs. The torsion stiffness of the slab connection lines is considered extremely high, in which case connection conditions become simpler. The boundary conditions in this case are obtained at the place $y_1=b_1$ for the main slab, as follows:

$$w_1(b)=0 \quad ; \quad \frac{\partial w_1}{\partial y}\bigg|_{y_1=b}=0 \tag{8}$$

Also, the boundary conditions at $y_2=0$ for the base slab are written as follows:

$$w_2(0)=0 \quad ; \quad \frac{\partial w_2}{\partial y}\bigg|_{y_2=0}=0 \tag{9}$$

Curvature functions are obtained to determine the proper solution of the problem by considering the connection conditions and the boundary such as the following. For the main slab:

$$w_1=\sum_m\sum_n B_{mn}^{(1)}\sin\lambda_m x\sin\mu_n y+\frac{1}{D_1}\left[F(y_1)N_m-\overline{N}_m\overline{F}(y_1)\right]\sin\lambda_m x \tag{10}$$

For the base slab is also written as follows:

$$w_2=\sum_m\sum_n B_{mn}^{(2)}\sin\lambda_m x\sin\mu_n y+\frac{1}{D_2}\left[F(y_2)N_m^*-\overline{N}_m^*\overline{F}(y_2)\right]\sin\lambda_m x \tag{11}$$

In which:

$$F(y_1)=\frac{b_1^2}{6}\left(y_{b_1}^3-3y_{b_1}^2+2y_{b1}\right) \quad ; \quad \overline{F}(y_1)=\frac{b_1^2}{6}\left(y_{b_1}^3-y_{b1}\right)$$

$$\lambda_m=\frac{m\pi}{a} \quad ; \quad y_{b_1}=\frac{y_1}{b_1} \quad ; \quad y_{b_2}=\frac{y_2}{b_2}$$

Therefore, considering the series coefficients used in problem solving, for the rectangular principal slab, the solution of the differential equation 1 is written as follows:

$$q(y)=q_o\cdot\frac{y_1}{b_1} \tag{12}$$

$$B_{mn}^{(1)}=-\frac{8\,q_o(-1)^n}{mn\pi^2 D_1\Delta_{mn}^2}-E_2(m_1 n)\left[N_m-(-1)^n\overline{N}_m\right] \tag{13}$$

In which:

$$m=1,3,5,\ldots \quad ; \quad n=1,2,3,\ldots.$$

Similarly, the solution of the differential equation 2 is written as follows:

$$q_2(y) = q_1\left(1 - \frac{y_2}{b_2}\right) \tag{14}$$

$$B_{mn}^{(2)} = -\frac{8\,q_1(-1)^n}{mn\pi^2 \cdot D_2 \cdot \overline{D}^*(m_1 n)} - E_2^*(m_1 n)\left[N_m^* - (-1)^n \overline{N}_m^*\right] \tag{15}$$

In which:

$$m = 1,3,5,\dots \quad ; \quad n = 1,2,3,\dots.$$

The signs and symptoms used in the calculations are as follows:

$$\Delta_{mn}^2 = \left(\lambda_m^2 + \mu_m^2\right)^2 \quad ; \quad \mu_n = \frac{n\pi}{b_1} \quad ; \quad \lambda_m = \frac{m\pi}{a}$$

$$D^*(m,n) = \Delta_{mn}^2 + \frac{K}{D_1} + \frac{K_s}{D_1}\Delta_{mn}$$

$$\overline{D}^*(m,n) = \left(\lambda_m^2 + \mu_n^2\right)^2\left[1 + \frac{K}{D_2}\cdot\frac{1}{\Delta_{mn}^2} + \frac{K_s}{D_2}\cdot\frac{1}{\Delta_{mn}}\right]$$

$$E_2(m,n) = \frac{2\,\lambda_m^2}{n\,\pi\,D_1\Delta_{mn}^2}\left(2 + \frac{\lambda_m^2}{\mu_n^2}\right)$$

$$E_2^*(m,n) = \frac{2\,\lambda_m^2}{n\,\pi\,D^*(m,n)}\left[2 + \frac{\lambda_m^2}{\mu_n^2} + \frac{K}{D_2\mu_n^2\lambda_m^2} + \frac{K_s}{D_2\mu_n^2}\left(1 + \frac{\lambda_m^2}{\mu_n^2}\right)\right]$$

According to solutions 10 and 11, the boundary conditions supplied for the main and base slabs are accepted as follows:

$$w_1(0) = 0 \quad ; \quad w_2(b_2) = 0 \tag{16}$$

By providing the first part of conditions 8, 9 and 16, the curvatures created at the site of the lines $y_1=0$ and $y_2=b_2$ also enter in solutions 10 and 11:

$$\begin{aligned} w_1 = \sum_m \sum_n B_{mn}^{(1)} \sin\lambda_m x \sin\mu_n y + \sum_m \Big\{\left[(1 - y_b) - \upsilon\lambda_m^2 F(y)\right]E_m + \\ + \left[y_{b_1} + \upsilon\lambda_m^2 \overline{F}(y)\right]\overline{E}_m + \frac{1}{D_1}\left[F(y)N_m - \overline{F}(y)\overline{N}_m\right]\Big\} \sin\lambda_m x \end{aligned} \tag{17}$$

$$\begin{aligned} w_2 = \sum_m \sum_n B_{mn}^{(2)} \sin\lambda_m x \sin\mu_n y + \sum_m \Big\{\left[(1 - y_{b_2}) - \upsilon\lambda_m^2 F(y)\right]E_m^* + \\ + \left[y_{b_2} + \upsilon\lambda_m^2 \overline{F}(y)\right]\overline{E}_m^* + \frac{1}{D_2}\left[F(y)N_m^* - \overline{F}(y)\overline{N}_m^*\right]\Big\} \sin\lambda_m x \end{aligned} \tag{18}$$

Problem solutions are presented as follows:

$$B_{mn}^{(1)} = -\frac{8\,q_o(-1)^n}{mn\pi^2 D_1\Delta_{mn}^2} - E_1(m_1 n)\left[E_m - (-1)^n \overline{E}_m\right] - E_2(m_1 n)\left[N_m - (-1)^n \overline{N}_m\right] \tag{19}$$

$$B_{mn}^{(2)} = -\frac{8\,q_1(-1)^n}{mn\pi^2 D_2 \cdot D^*(m_1 n)} - E_1^*(m_1 n)\left[E_m^* - (-1)^n \overline{E}_m^*\right] - E_2^*(m_1 n)\left[N_m^* - (-1)^n \overline{N}_m^*\right] \tag{20}$$

$$E_2(m,n)=\frac{2\lambda_m^4}{n\pi D(m,n)}\left(1-\upsilon\frac{\lambda_m^2}{\mu_n^2}-2\upsilon\right) \tag{21}$$

$$E_1^*(m,n)=\frac{2D_2\lambda_m^4}{n\pi D^*(m,n)}\times\left[1-\upsilon\frac{\lambda_m^2}{\mu_n^2}-2\upsilon+\frac{K}{D_2\mu\lambda_m^4}\left(1-\upsilon\frac{\lambda_m^2}{\mu_n^2}\right)+\frac{K_s}{D\lambda_n^2}\left(1-\upsilon\frac{\lambda_m^2}{\mu_n^2}-\upsilon\right)\right] \tag{22}$$

The coefficients of the imposed series in solutions 17 and 18 of the problem will be determined according to the connection conditions and the boundary of the problem. Considering hinged joint at the site $y_1=0$, the main slab and $y_2=b_2$ the base slab with the front slab of the boundary conditions are obtained in solutions 17 and 18, are as follows:

$$E_m=\overline{E}_m=0 \quad ; \quad E_m^*=\overline{E}_m^*=0$$

Additionally, with zeroing the bending moment in the hinged joining of the main and base slab with a front slab, $N_m=0$ and $\overline{N}_m^*=0$ are provided. Therefore, the unknown quantities are merely coefficients of Fourier series N_m and $\overline{N}_m^*$ separated from the bending moments created at the location of the connecting lines of the two flat slabs, which is required to determine the connection conditions of 5, 6 and 7. Considering the torsion stiffness at the connection point of the slabs in the form $C=\infty$, the connection conditions are obtained as follows:

$$\left.\frac{\partial w_1}{\partial y}\right|_{y_1=b_1}=0 \quad ; \quad \left.\frac{\partial w_2}{\partial y}\right|_{y_2=b_2}=0 \tag{23}$$

In this case, the solving of the main and base slabs is not interdependent and they are separated into two independent issues. The problem-solving algorithm is considered generically. In this case, it is necessary to pay attention to the algebraic equations obtained by considering the connection conditions separately. By using solutions 10 and 11, the connection conditions 23 are considered in algebraic equations. Therefore, we have:

$$\left.\frac{\partial w_1}{\partial y}\right|_{y=b}=\sum_m\sum_n B_{mn}^{(1)}\mu_n\sin\lambda_m x\cos\mu_n y\Big|_{y=b}+$$
$$+\frac{1}{D_1}\sum_m\left[\frac{b_1}{6}\left(3y_b^2-6y_b+2\right)N_m(0)-\frac{b_1}{6}\left(3y_b^2-1\right)\overline{N}_m(1)\right]\sin\lambda_m x\Big|_{y=b}$$

$$\left.\frac{\partial w_2}{\partial y}\right|_{y=0}=\sum_m\sum_n B_{mn}^{(2)}\mu_n\sin\lambda_m x\cos\mu_n y\Big|_{y=0}+$$
$$+\frac{1}{D_2}\sum_m\left[\frac{b_2}{6}\left(3y_b^2-6y_b+2\right)\overline{N}_m^*(1)-\frac{b_2}{6}\left(3y_b^2-1\right)N_m^*(2)\right]\sin\lambda_m x\Big|_{y=0}$$

$$\left.\frac{\partial^2 w_1}{\partial x^2}\right|_{y=b}=-\sum_m\sum_n B_{mn}^{(1)}\lambda_m^2\sin\lambda_m x\sin\mu_n y\Big|_{y=b}-$$
$$-\sum_m\frac{1}{D_1}\left[F(y)N_m(0)-\overline{F}(y)N_m(1)\right]\lambda_m^2\sin\lambda_m x\Big|_{y=b}$$

$$\left.\frac{\partial^2 w_1}{\partial y^2}\right|_{y=b}=-\sum_m\sum_n B_{mn}^{(1)}\mu_m^2\sin\lambda_m x\sin\mu_n y\Big|_{y=b}+$$
$$+\frac{1}{D_1}\sum_m\left[\left(y_{b_1}-1\right)N_m(0)-y_{b_1}N_m(1)\right]\sin\lambda_m x\Big|_{y=b}$$

$$\left.\frac{\partial^3 w_1}{\partial x^2 \partial y}\right|_{y=b} = -\sum_m \sum_n B_{mn}^{(1)} \lambda_m^2 \mu_n \sin \lambda_m x \cos \mu_n y \Big|_{y=b} -$$

$$-\sum_m \lambda_m^2 \left[\frac{b_1}{6} \left(3y_b^2 - 6y_b + 2\right) N_m(1) - \frac{b_1}{6} \left(3y_b^2 - 1\right) N_m(1) \right] \sin \lambda_m x \Big|_{y=b}$$

$$\left.\frac{\partial^2 w_2}{\partial x^2}\right|_{y=0} = -\sum_m \sum_n B_{mn}^{(2)} \lambda_m^2 \sin \lambda_m x \sin \mu_n y \Big|_{y=0} -$$

$$-\sum_m \frac{1}{D_2} \left[F(y) N_m^*(1) - \overline{F}(y) \overline{N}_m(2) \right] \lambda_m^2 \sin \lambda_m x \Big|_{y=0}$$

$$\left.\frac{\partial^2 w_2}{\partial y^2}\right|_{y=0} = -\sum_m \sum_n B_{mn}^{(2)} \mu_m^2 \sin \lambda_m x \sin \mu_n y \Big|_{y=0} +$$

$$+\sum_m \frac{1}{D_2} \left[\left(y_{b_2} - 1\right) N_m^*(1) - y_{b_2} N_m^*(2) \right] \sin \lambda_m x \Big|_{y=0}$$

With respect to the values of $y=0$ and $y=b$ in the functions and based on the principle of equilibrium and considering the connection conditions, the following equations system is obtained. With condition $\left.\frac{\partial w_1}{\partial y}\right|_{y=b} - \left.\frac{\partial w_2}{\partial y}\right|_{y=0} = 0$ we will have:

$$-\sum_n B_{mn}^{(1)} \mu_n + \frac{1}{D_1} \left[-\frac{b_1}{6} N_m(0) - \frac{b_1}{3} N_m(1) \right] - \sum_n B_{mn}^{(2)} \mu_n - \frac{1}{D_2} \left[\frac{b_2}{3} N_m^*(1) + \frac{b_2}{6} \overline{N}_m^*(2) \right] = 0$$
(24)

$$C \left.\frac{\partial^3 w_1}{\partial x^2 \partial y}\right|_{y=b} = D_1 \left(\frac{\partial^2 w_1}{\partial y^2} + \upsilon \frac{\partial^2 w_1}{\partial x^2} \right)\Big|_{y=b} - D_2 \left(\frac{\partial^2 w_2}{\partial y^2} + \upsilon \frac{\partial^2 w_2}{\partial x^2} \right)\Big|_{y=0} \quad (25)$$

Therefore, the following algebraic equation is obtained:

$$C \left\{ \sum_n B_{mn}^{(1)} \lambda_m^2 \mu_n - \lambda_m^2 \left[\frac{b_1}{6\, D_1} N_m(0) + \frac{b_1}{3\, D_1} \overline{N}_m(1) \right] \right\} = -N_m(1) + N_m(1) \quad (26)$$

With the replacement of solutions 19 and 20 in equations 24 and 26, the following algebraic equations are obtained:

$$d_m(0) N_m(0) + d_m(1) N_m(1) + \overline{d}_m(1) \overline{N}_m(1) + d_m(2) N_m(2) = \delta_m \quad (27)$$

$$N_m(1) - \overline{N}_m(1) = \frac{\delta_c}{\gamma} m^2 \pi^2 \left[\delta_m^* - d_m^{(0)} N_m(0) - d_m^{(1)} N_m(1) \right] \quad (28)$$

The signs and symptoms used in the calculations are as follows:

$$d_m(0) = \left[-\frac{1}{6} + \sum_n n\pi E_2(m,n) \right] \frac{b_1^2}{D_1} \quad ; \quad d_m(1) = \left[-\frac{1}{3} - \sum_n (1)^n n\pi E_2(m,n) \right] \frac{b_1^2}{D_1}$$

$$\overline{d}_m(1) = \left[-\frac{1}{3} + \sum_n n\pi E_2^*(m,n) \right] \frac{b_2^2}{D_2} \quad ; \quad d_m(2) = \left[-\frac{1}{6} - \sum_n (-1)^n n\pi E_2^*(m,n) \right] \frac{b_2^2}{D_2}$$

$$\delta_m = -\frac{8}{m\pi} \sum_n \left(\frac{q_o}{D_1 \Delta_{mn}^2} + \frac{q_1}{D_2 \cdot D^*(m,n)} \right) (-1)^n$$

$$\delta_c = \frac{C}{aD_1} \quad ; \quad \gamma = \frac{a}{b_1} \quad ; \quad \delta_m^* = -\frac{8\,q_o}{m\pi}\sum_n \left(-1\right)^n \frac{1}{\Delta_{mn}^2}$$

In this way, solving the problem leads to the solution of the algebraic equations system of 27 and 28. The analysis of these equations shows that the equations are similar to those obtained for the equation of three moments of continuous beams, which proves the correct solution of the problem. In practical calculations, it is possible to ignore the resistant elements in the location of the slab connection lines, in which case it will be $\delta_c = 0$. In this case, we will have equation 28:

$$N_m\left(1\right) = \overline{N}_m\left(1\right)$$

It also equation 27 as follows:

$$d_m(0)N_m(0) + \left[d_m(1) + \overline{d}_m(1)\right]N_m(1) + d_m(2)N_m(2) = \delta_m \tag{29}$$

Considering the hinged joint between the main slab with the front slab and the base slab with the front slab (point 2), $N_m\left(0\right) = N_m\left(2\right) = 0$ and the number of unknown issues is also reduced. So we'll have:

$$\begin{aligned} &d_m(1)N_m(1) + \overline{d}_m(1)\overline{N}_m(1) = \delta_m \\ &N_m(1) - \overline{N}_m(1) = \frac{\delta_c}{\gamma} m^2\pi^2\left[\delta_m^* - d_m(1)N_m(1)\right] \end{aligned} \tag{30}$$

By disregarding the resistant elements in the location of the connecting lines, $\delta_c = 0$, and solving the problem is as follows:

$$N_m(1) = \overline{N}_m(1) = \frac{\delta_m}{\left[d_m(1) + \overline{d}_m(1)\right]} \tag{31}$$

As the function of flexures is known, the bending moments created in the main slab are determined by the following relationships:

$$\begin{aligned} M_x &= -D_1\left(\frac{\partial^2 w_1}{\partial x^2} + \upsilon\frac{\partial^2 w_1}{\partial y^2}\right) = \sum_m \sum_n D_1\left(\lambda_m^2 + \upsilon\mu_n^2\right)B_{mn}^{(1)} \sin\lambda_m x \sin\mu_n y + \\ &+ \sum_m \left\{\left[F(y)\lambda_m^2 - \upsilon(y_b - 1)\right]N_m(0) - \left[\lambda_m^2\overline{F}(y) - \upsilon y_{b_1}\right]N_m(1)\right\} \times \sin\lambda_m x \end{aligned}$$

$$\begin{aligned} M_y &= -D_1\left(\frac{\partial^2 w_1}{\partial y^2} + \upsilon\frac{\partial^2 w_1}{\partial x^2}\right) = \sum_m \sum_n D_1\left(\mu_n^2 + \upsilon\lambda_m^2\right)B_{mn}^{(1)} \sin\lambda_m x \sin\mu_n y - \\ &- \sum_m \left\{\left[(y_b - 1) - \upsilon F(y)\lambda_m^2\right]N_m(0) - \left[y_b\lambda_m^2 - \upsilon\overline{F}(y) - \lambda_m^2\right]N_m(1)\right\} \times \sin\lambda_m x \end{aligned}$$

Considering $a = b_1$ and $a = b_2$ in the retaining wall, the problem-solving algorithm is as follows:

$$E_1(m,n) = \frac{2\,m^4}{n\pi\left(m^2 + n^2\right)^2}\left(1 - \upsilon\frac{m^2}{n^2} - 2\,\upsilon\right)$$

$$E_2(m,n) = \frac{2\,m^2}{n\pi^3\left(m^2 + n^2\right)^2}\left(2 + \frac{m^2}{n^2}\right)$$

$$E_1^*(m,n) = \frac{2\,m^4}{n\pi\left(m^2 + n^2\right)^2\eta(m,n)}\left[1 - \upsilon\frac{m^2}{n^2} - 2\,\upsilon + \overline{K}\left(1 - \upsilon\frac{m^2}{n^2}\right) + \overline{K}_s\left(1 - \upsilon\frac{m^2}{n^2} - \upsilon\right)\right]$$

$$E_2^*(m,n) = \frac{2\,m^2}{n\pi^3\left(m^2 + n^2\right)^2\eta(m,n)} \times \left[12 + \frac{m^2}{n^2} + \frac{\overline{K}}{m^2n^2} + \frac{\overline{K}_s}{n^2}\left(1 + \frac{m^2}{n^2}\right)\right]$$

$$\eta(m,n)=\left[1+\frac{\overline{K}}{(m^2+n^2)^2}+\frac{\overline{K}_s}{m^2+n^2}\right]$$

$$\overline{K}=\frac{Ka^4}{D\pi^4}\quad;\quad \overline{K}_s=\frac{K_s a^2}{D\pi^2}\quad;\quad \Delta^2_{mn}=(m^2+n^2)^2$$

The quantities used in problem solving are as follows. K and K_s are elastic characteristics of the soil. K is the compressive stiffness coefficient of the soil and K_s is the slip stiffness coefficient of the soil. Also, soil with a moderate density is considered.

$$K=5{,}0\,\frac{kg}{cm^3}\quad;\quad K_s=Ka=4435\,\frac{kg}{cm^3}\quad;\quad \frac{a}{h}=30\quad;\quad a=600\,cm\quad;\quad h=20\,cm\quad;\quad E_b=290000\,\frac{kg}{cm^2}$$

$$\overline{K}=\frac{Ka^4}{D\pi^4}=\frac{12(1-\upsilon^2)Ka\cdot a^3}{E\cdot\pi^4 h^3}=\frac{12(1-0{,}17^2)\cdot 5\cdot 600\cdot 30^3}{2{,}9\cdot 10^5\pi^4}=33{,}42\quad;\quad \overline{K}_s=\frac{K_s a^2}{D\pi^2}==\frac{4435\cdot 12(1-\upsilon^2)\cdot 30^3}{2{,}9\cdot 10^5\cdot 600\cdot\pi^2}=0{,}818$$

3. Results and discussion

Considering the above values, the solution of rectangular slabs sitting on the elastic bed is analyzed. Thus, the coefficients entered in the problem solving are calculated and presented in Tables 1 through 6.

Table 1: Amount of Δ^2_{mn}

m \ n	1	2	3
1	4	25	100
3	100	169	324

Table 2: Amount of $\eta(m,n)$

m \ n	1	2	3
1	9,39	2,353	1,342
3	1,342	1,203	1,123

Table 3: Amount of $E_1(m,n)$

m \ n	1	2	3
1	0,078	0,00823	0,00561
3	- 0,459	0,0476	0,0201

Table 4: Amount of $E_1^*(m,n)$

m \ n	1	2	3
1	0,478	0,184	0,00104

3	- 3,439	2,652	1,187

Table 5: Amount of $E_2(m,n)$

m \ *n*	1	2	3
1	0,0477	0,00286	0,000448
3	0,0187	0,00455	0,00136

Table 6: Amount of $E_2^*(m,n)$

m \ *n*	1	2	3
1	0,0603	0,00342	0,000375
3	0,0628	0,00448	0,00179

Using these auxiliary parameters, the coefficients of the algebraic equation system are calculated:

$$d_1(1) = -\frac{1}{3} - \sum_m (-1)^n n\pi E_2(m,n) = -0{,}333 + 3{,}1415(0{,}0477 - 2\cdot 0{,}00289 + 3\cdot 0{,}000448) = -0{,}333 + 0{,}136 = -0{,}197;$$

$$\bar{d}_1(1) = -\frac{1}{3} + \sum_m n\pi E_2^*(m,n) = -0{,}333 + 3{,}1415(0{,}0603 + 2\cdot 0{,}00342 + 3\cdot 0{,}000975) = -0{,}333 + 0{,}220 = -0{,}113$$

$$\delta_1 = -\frac{8}{m\pi}\sum_m \left(\frac{q_o}{\pi^4(m^2+n^2)^2} + \frac{q_1}{\pi^4(m^2+n^2)^2\eta(m,n)}\right)(-1)^n = -\frac{8}{\pi}[(-0{,}00257 + 0{,}00041 - 0{,}0001)q_o + (-0{,}00273 + 0{,}000172 - 0{,}000077)q_1] = 0{,}00628\frac{q_o a^4}{D_1}$$

$$\delta_3 = -\frac{8}{3\pi}(-0{,}00010 + 0{,}000061 - 0{,}0000316)q_o + (-0{,}000075 + 0{,}000047 - 0{,}000068)q_1 = \frac{8q_o}{3\pi}(0{,}000478 + 0{,}000290) = 0{,}0000652\frac{q_o a^4}{D_1}$$

$$d_3(1) = -0{,}333 + 3{,}14156(0{,}0187 - 2\cdot 0{,}00455 + 3\cdot 0{,}00136) = -0{,}333 + 0{,}043 = -0{,}29$$

$$\bar{d}_3(1) = -0{,}333 + 3{,}14156(0{,}0628 + 2\cdot 0{,}00448 + 3\cdot 0{,}00179) = -0{,}333 + 0{,}242 = -0{,}091$$

$$\delta_c = \frac{C}{aD_1} = \frac{GJ_p\cdot 12(1-\upsilon^2)}{Eh^3\cdot a} = \frac{E\cdot\alpha\, b_o^3 h_o\cdot 12(1-\upsilon^2)}{2(1+\upsilon)Eh^3\cdot a}$$

$$\delta_c = \frac{C}{aD_1} = \frac{6(1-\upsilon)\alpha\, b_o^3 h_o}{ah^3} = \frac{6(1-0{,}17)\cdot 0{,}209\cdot 30^3}{60} = 15{,}6$$

$$\gamma = \frac{a}{b_1} = 1 \quad ; \quad \upsilon = 0{,}17$$

$$\delta_m^* = -\frac{8\,q_o}{m\pi}\sum_n (-1)^n \frac{1}{\pi^4(m^2+n^2)^2}$$

$$\delta_1^* = -\frac{8\,q_o}{\pi^5}(0{,}25 + 0{,}04 - 0{,}01) = 0{,}00576$$

$$\delta_3^* = -\frac{8\,q_o}{3\cdot\pi^5}(0{,}01+0{,}00591-0{,}00308)=0{,}0000625$$

Considering the above values, the algebraic equation system of the problem is written as below. If we consider $m=1$, we will have:

$$-0{,}197\frac{a^2}{D}N_1(1)-0{,}113\frac{a^2}{D}N_1(1)=0{,}00628\frac{qa^4}{D}$$

$$N_1(1)-\overline{N}_1(1)=154{,}0(0{,}00576+0{,}197N_1(1))$$

As a result:

$$N_1(1)=-0{,}03006qa^2 \quad ; \quad \overline{N}_1(1)=-0{,}00501qa^2$$

If we consider $m=3$, we will have:

$$-0{,}29\frac{a^2}{D}N_3(1)-0{,}091\overline{N}_3(1)=0{,}00652\frac{qa^4}{D}$$

$$N_3(1)-\overline{N}_3(1)=1386(0{,}0000625+0{,}29N_3(1))$$

As a result:

$$N_3(1)=-0{,}00204qa^2 \quad ; \quad \overline{N}_3(1)=-0{,}000491qa^2$$

The doubles series coefficients of the problem are determined using relations 13 and 15.

$$B_{mn}^{(1)}=\left[-\frac{8\,q_o(-1)^n}{mn\pi^6(m^2+n^2)^2}+E_2(m,n)(-1)^n N_m(1)\right]\frac{qa^4}{D}$$

Table 7: Amount of $B_{mn}^{(1)}$

m \ n	1	2	3	$N_m(1)$
1	0,00351	0,000252	0,0000412	- 0,0306
3	0,0000658	0,0000178	0,00000562	- 0,00204

If the main slab connection to the bottom slab is considered to be hinged, then the coefficients of the problem series for the main slab are calculated as follows, and are given in Table 8:

$$B_{mn}^{(1)}=-\frac{8\,q_o(-1)^n}{mn\pi^6(m^2+n^2)^2}\cdot\frac{q_o a^4}{D}$$

Table 8: Amount of $B_{mn}^{(1)}$

m \ n	1	2	3
1	0,00208	- 0,000166	0,0000277
3	0,0000277	- 0,0000082	0,00000285

After determining the series coefficients, the flexures and bending moments created in the main slab are calculated using the following relationships:

$$w_1 = \sum_m \sum_n B_{mn}^{(1)} \sin\lambda_m x \sin\mu_n y - \frac{1}{D_1}\sum_m N_m(1)\overline{F}(y)\sin\lambda_m x$$

$$M_y = D_1 \sum_m \sum_n \left(n^2 + \upsilon m^2\right) B_{mn}^{(1)} \sin\lambda_m x \sin\mu_n y + \sum_m \left[y_b - \upsilon \overline{F}(y)\lambda_m^2\right] N_m(1)\Big\} \sin\lambda_m x$$

The values obtained for the flexures and bending moments created in the main slab at the cutting location $x = 0,5a$ are written in Table 9.

Table 9: Amount of flexures and bending moments.

y_b	$10^{-2} w \frac{qa^4}{D}$	10^{-2} My qa^2
0,0	0,0	0,0
0,1	0,877	1,05
0,2	1,798	1,48
0,3	1,283	2,089
0,4	1,563	2,500
0,5	1,690	2,600
0,6	1,617	2,331
0,7	1,330	1,670
0,8	0,900	0,443
0,9	0,400	- 1,10
1,0	0,0	- 2,80

The coefficients of the problem series for the base slab are also calculated as follows, and are given in Table 10:

$$B_{mn}^{(2)} = -\frac{8\, q_o (-1)^n}{mn\pi^6 \left(m^2 + n^2\right)^2 \eta_{mn}} - E_2^*(m,n)\overline{N}_m(1)$$

Table 10: Amount of $B_{mn}^{(2)}$

m \ n	1	2	3	$\overline{N}_m(1)$
1	0,000523	+ 0,0000876	0,0000255	- 0,00501
3	0,0000236	0,0000702	0,00000343	- 0,00049

Considering the hinged joints between the main slab with the base slab, the series coefficient of problem for the base slab is calculated in Table 11:

Table 11: Amount of $B_{mn}^{(2)}$

m* \ *n	1	2	3
1	0,000221	+ 0,0000705	0,0000206
3	0,0000206	+ 0,00000681	0,00000255

The values obtained for the flexures and bending moments created in the base slab are written in Table 12 in relation to the values given in Table 9 at the cutting location $x = 0,5a$.

Table 12: Amount of flexures and bending moments.

y_b	$10^{-3} w \frac{qa^4}{D}$	$10^{-2} My\, qa^2$
0,0	0,0	- 0,452
0,1	0,036	- 0,099
0,2	0,092	0,182
0,3	0,204	0,347
0,4	0,216	0,384
0,5	0,2170	0,317
0,6	0,214	0,105
0,7	0,812	0,0784
0,8	0,132	- 0,0036
0,9	0,0225	- 0,0162
1,0	0,0	0,0

Comparison of the results in tables 7, 8, 10, and 11, as well as tables 9 and 12 show that the base slab stiffness due to being placed on the elastic bed is ten times less than the main flat slab against the soil pressure, and the flexures and bending moments is also eight to ten times less. If the bottom slab thickness is considered to be relatively high relative to the main slab thickness, its flexural rigidity will increase. To simplify problem solving, the main slab connection with the base slab is considered to be rigid and restrained. In this case, the boundary conditions and connection are as follows. The boundary condition of the main slab at the location of the line $y_1=b_1$ is determined as follows:

$$w_1(b_1) = 0 \quad ; \quad \left.\frac{\partial w_1}{\partial y}\right|_{y=b_1} = 0$$

Using these conditions, the following algebraic equations system is obtained:

$$-\frac{1}{D}\frac{b_1}{3}N_m(1)=\sum_n \mu_n B_{mn}^{(1)} \quad ; \quad \left[-\frac{b_1}{3D_1}-\sum_n n\pi(-1)^n E_2(m,n)\right]\cdot N_m(1)=-\sum_n \frac{8\,q_o(-1)^n}{m\pi D_1\Delta_{mn}^2} \quad ; \quad d_1(m)\,N_m(1)=\delta_m$$

In this way, the problem is solved:

$$N_{(m)}(1)=\frac{8\,q\,a^2(-1)^n}{\left(-\frac{1}{3}-\sum_n n\pi E_2(m,n)\right)}\cdot\frac{1}{mn^5\left(m^2+n^2\right)^2} \tag{32}$$

Also, the series coefficients are obtained as follows:

$$N_1(1)=-0{,}03001qa^2 \quad ; \quad N_3(1)=-0{,}00215\cdot q_o a^2$$

The results show that the base slab stiffness is much larger than the main slab. Similarly, for the base slab, taking into account the rigid connection at the $y_2=0$ line, the following solution is obtained:

$$\overline{N}_{(m)}(1)=-\frac{8\,q_o\,a^2}{-\frac{1}{3}+\sum_n n\pi E_2^*(m,n)}\cdot\frac{1}{mn^5\,\eta_{mn}\cdot\left(m^2+n^2\right)^2} \tag{33}$$

Finally, it can be stated that in order to calculate the proposed wall, it is possible to connect the slabs in the apex of the wall as hinged joints and connect the upper slabs with base slab in a rigid connection.

Conclusion

The use of plate structures in the proposed new structure for the concrete gravity retaining walls due to the considerable savings in concrete consumption (about 90%) has given special attention to this research. On the other hand, for the first time, the calculation and analysis of a new type of structural form of slab-shaped concrete gravity retaining wall is presented.

There are various methods for analyzing the plates, how each of these principles is applied depends on the geometric shape, plate size, plate application, and boundary conditions as well as the loads involved. In order to analyze and calculate the new wall of flat slabs, it is necessary to have equations that, while providing the required mechanical properties of the structure with precision, can also be solved mathematically. In order to achieve this, the proposed method in this study can be powerful. In this paper, mathematics explicitly provides a physical understanding of the behavior of the plates in the proposed structure, and the solved problem has an effective role in attracting and understanding the main concepts and their practical application.

The bending of the rectangular plate of the structure provided in this problem has led to solutions in the form of the series. In other words, flexures and moments are expressed and solved in series. Also, with increasing number of phrases in the series, the accuracy has increased.

Comparison of the results in the tables of this study shows that the base flat slab stiffness due to being placed on the elastic bed is 10 times less than that of the main flat slab against the soil pressure, and the flexures and bending moments are eight to ten times lower. In addition, by increasing the thickness of the bottom slab, the flexural rigidity increases. In other words, as the base slab thickness increases, the calculation of the slab is considered to be quite rigid.

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